RATIO AND PROPORTION

EQUIVALENT RATIOS

Orange squash is to be mixed with water in a ratio of 1 : 6 ; this means that for every unit of orange squash, 6 units of water will be used. The table gives some examples:

Amount of Orange Squash (cm^3)	Amount of water (cm^3)
1	6
20	120
5	30

The ratios 1 : 6 and 20 : 120 and 5 : 30 are all equivalent ratios, but 1 : 6 is the simplest form.

Ratios can be simplified by dividing both sides by the same number : note the similarity to fractions. An alternative method for some purpose, is to reduce to the form 1 : n or n : 1 by dividing both numbers by either the left-hand-side (LHS) or the right-hand-side (RHS). For example :

the ratio 4 : 10 may be simplified to

$$\frac{4}{4}:\frac{10}{4} \Rightarrow 1:25$$

the ratio 8 : 5 may be simplified to

$$\frac{8}{5}:\frac{5}{5} \Rightarrow 1.6:1$$

Ex.1 Write each of these ratios in its simplest form :

(a) 7 : 14 (b) 15 : 25

(c) 10 : 4

Sol.

(a) Divide both sides by 7, giving

$$7:14 = \frac{7}{7}:\frac{14}{7} = 1:2$$

(b) Divide both sides by 5, giving

$$15:25 = \frac{15}{5}:\frac{25}{5} = 3:5$$

(c) Divide both sides by 2, giving

$$10:4 = \frac{10}{2}:\frac{4}{2} = 5:2$$

Ex.2 Write these ratios in the fom 1 : n.

(a) 3 : 12 (b) 5 : 6 (c) 10 : 42

Sol.

(a) Divide both sides by 3, giving

3 : 12 = 1 : 4

(b) Divide bothe sides by 5, giving

$$5:6 = 1:\frac{6}{5} = 1:1.2$$

(c) Divide both sides by 10, giving

$$10:42 = 1:\frac{42}{10} = 1:4.2$$

Ex.3 The scale on a map is 1 : 20000. What actual distance does a length of 8cm on the map represent ?

Sol. Actual distance = 8 × 20000

= 160 000 cm = 1600 m = 1.6 km

DIRECT PROPORTION

Direct proportion can be used to carry out calculations like the one below:

If 10 calculators cost £ 120,

then 1 calculator costs £ 12,

And 8 calculators cost £ 96.

Ex.4 If 6 copies of a book cost £ 9, calculate the cost of 8 books.

Sol. If 6 copies cost £ 9,

then 1 copy costs £ $\frac{9}{6}$ = £ 1.50

and 8 copies cost £ 1.50 × 8 = £ 12

Ex.5 If 25 floppy discs cost £ 5.50, calculate the cost of 11 floppy discs.

Sol. If 25 discs cost £ 5.50 = 550p

then 1 disc costs = $\frac{550}{25}$ = 22p

so 11 discs cost 11 × 22p = 242 p = £ 2.42

PROPORTIONAL DIVISION

Sometimes we need to divide something in a given ratio. Malcolm and Alison share the profits from their business in the ratio 2 : 3. This means that, out of every £ 5 profit. Malcolm gets £ 2 and Alison gets £ 3.

Ex.6 Julie and Jack run a stall at a car boot sale and take a total of £ 90. They share the money in the ratio 4 : 5. How much money does each receive.

Sol. As the ratio is 4 : 5, first add these numbers together to see by how many parts the £ 90 is to be divided.

4 + 5 = 9, so 9 parts are needed.

Now divide the total by 9.

$\frac{90}{9}$ = 10, so each part is £10.

Julie gets 4 parts at £ 10, giving 4 × £ 10 = £ 40.

Jack gets 5 parts at £ 10, giving 5 × £ 10 = £ 50.

Ex.7 Rachel, Ben and Emma are given £ 52. They decide to divide the money in the ratio of their ages, 10 : 9 : 7. How much does each receive ?

Sol. 10 + 9 + 7 = so 26 parts are needed.

Now divide the total by 26.

$\frac{52}{26}$ = 2, so each part is £ 2.

Rachel gets 10 parts at £ 2, giving

10 × £ 2 = £ 20

Ben gets 9 parts at £ 2, giving

9 × £ 2 = £ 18

Emma gets 7 parts at £ 2, giving

7 × £ 2 = £ 14

LINEAR CONVERSION

The ideas used in this unit can be used for converting masses, lengths and currencies.

Ex.8 If £ 1 is worth 9 French francs, convert :

(a) £ 22 to Ff, (b) 45 Ff to £

(c) 100 Ff to £.

Sol.

(a) £ 22 = 22 × 9 = 198 Ff

(b) 1 Ff = £ $\frac{1}{9}$ so 45 Ff = 45 × $\frac{1}{9}$ = $\frac{45}{9}$ = £ 5

(c) 100 Ff = 100 × $\frac{1}{9}$ = $\frac{100}{9}$ = £11 $\frac{1}{9}$

= £ 11.11 to the nearest pence.

Ex.9 Use the fact that 1 foot is approximately 30 cm to convert :

(a) 8 feet to cm

(b) 50 cm to feet

(c) 195 cm to feet

Sol.

(a) 8 feet = 8 × 30 = 240 cm

(b) 1 cm = $\frac{1}{30}$ feet, so 50 cm = 50 × $\frac{1}{30}$

$= \frac{5}{3} = 1\frac{2}{3}$ feet

(c) 195 cm = 195 × $\frac{1}{30} = \frac{195}{30} = \frac{13}{2} = 6\frac{1}{2}$ feet

Ex.10 If £ 1 is worth $ 1.60, convert :

(a) £ 15 to dollars

(b) $ 8 to pounds

Sol.

(a) £ 15 = 15 × 1.60 = $ 24.

(b) $ 1 = £ $\frac{1}{1.60}$ = £ $\frac{10}{16}$

$ 8 = 8 × $\frac{10}{16} = \frac{80}{16}$ = £ 5

INVERSE PROPORTION

Inverse proportion is when an increase in one quantity causes a decrease in another:

The relationship between speed and time is an example of inverse proportionality : as the speed increases, the journey time decreases, so the time for a journey can be found dividing the distance by the speed.

Ex.11

(a) Ben rides his bike at a speed of 10 mph. How long does it takes him to cycle 40 miles ?

(b) On another day he cycles the same route at a speed of 16 mph. How much time does this journey take ?

Sol.

(a) Time = $\frac{40}{10}$ = 4 hours

Note : Faster speed ⇒ shorter time.

(b) Time = $\frac{40}{16} = 2\frac{1}{2} = 2\frac{1}{2}$ hours .

Ex.11 Jai has to travel 280 miles. How long does it take if he travels at :

(a) 50 mph

(b) 60 mph

(c) How much time does he save when he travels at the faster speed ?

Sol.

(a) Time = $\frac{280}{50}$ = 5.6 hours = 5 hours 36 minutes.

(b) Time = $\frac{280}{60} = 4\frac{2}{3}$ hours = 4 hours 40 minutes

(c) Time saved = 5 hours 36 mins – 4 hours 40 mins = 56 minutes

Ex.12 In a factory, each employee make 40 chicken pies in one hour. How long will it take :

(a) 6 people to make 40 pies,

(b) 3 people to make 240 pies,

(c) 10 people to make 600 pies ?

Sol.

(a) 1 person makes 40 pies in 1 hour.

6 people make 40 pies in $\frac{1}{6}$ hour (or 10 minutes).

(b) 1 person makes 40 pies in 1 hour

1 person makes 240 pies in $\frac{240}{40}$ = 6 hours.

3 people make 240 pies in $\frac{6}{3}$ = 2 hours.

(c) 1 person makes 40 pies in 1 hour.

1 person makes 600 pies in $\frac{600}{40}$ = 15 hours.

10 people make 600 pies in $\frac{15}{10} = 1\frac{1}{2}$ hours.

IMPORTANT FACTS & FORMULAE

1. **Ratio :** The ratio of two quantities a & b in the same units, is the fraction $\frac{a}{b}$ and we write it as a:b.

In the ratio a : b, we call a as the first term or antecedent and b, the second term or consequent.

Ex. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule : The multiplication or division of each term of a ratio by the same nor-zero number does not affect the ratio.

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2:3.

2. **Proportion :** The equality of two ratios is called proportion.

If a : b = c : d, we write, a : b : : c : d and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.

Product of means : Product of extremes.

Thus, a : b : : c : d $\Leftrightarrow$ (b × c) = (a × d).

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2 : 3.

3. **(i)** **Fourth Proportional :** If a : b = c : d, then d is called the fourth proportional to a, b, c.

(ii) **Third Proportional :** If a : b = b : c, then c is called the third proportional to a and b.

(iii) **Mean Proportional :** Mean proportional between a and b is $\sqrt{ab}$.

4. **(i)** **Comparison of Ratios :**

We say that (a : b) > (c : d) $\Leftrightarrow \frac{a}{b} > \frac{c}{d}$

(ii) **Compounded Ratio :** The compounded ratio of the ratios (a : b), (c : d), (e : f) is (ace : bdf).

5. **(i)** **Duplicate ratio** of (a : b) is $(a^2 : b^2)$

(ii) **Sub-duplicate ratio:**

(a : b) is $(\sqrt{a} : \sqrt{b})$.

(iii) **Triplicate ratio:** of (a : b) is $(a^3:b^3)$.

(iv) **Sub-Triplicate ratio:** of (a : b) is $(a^{1/3} : b^{1/3})$.

(v) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

(Componendo and Dividendo)

6. **Variation**

(i) We say that x is directly proportion to y, if x = ky for some constant k and we write, $x \propto y$.

(ii) We say that x is inversely proportional to y, if xy = k for some. constant k and we write, $x \propto \frac{1}{y}$.

WORKSHEET

1. If A : B = 5 : 7 and B : C = 6 : 11, then A : B : C is :

(a) 55 : 77 : 66 (b) 30 : 42 : 77

(c) 35 : 49 : 42 (d) None of these

2. If A : B = 3 : 4 and B : C = 8 : 9, then A : C is :

(a) 1 : 3 (b) 3 : 2 (c) 2 : 3 (d) 1 : 2

3. If A : B = 8 : 15, B : C = 5 : 8 and C : D = 4 : 5, then A : D is equal to :

(a) 2 : 7 (b) 4 : 15 (c) 8 : 15 (d) 15 : 4

4. If A : B : C = 2 : 3 : 4, then $\frac{A}{B}:\frac{B}{C}:\frac{C}{A}$ is equal to :

(a) 4 : 9 : 16 (b) 8 : 9 : 12

(c) 8 : 9 : 16 (d) 8 : 9 : 24

5. If A : B = $\frac{1}{2}:\frac{3}{8}$, B : C $\frac{1}{3}:\frac{5}{9}$= and C : D = $\frac{5}{6}:\frac{3}{4}$, the the ratio A : B : C : D is :

(a) 4 : 6 : 8 : 10 (b) 6 : 4 : 8 : 10

(c) 6 : 8 : 9 : 10 (d) 8 : 6 : 10 : 9

6. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 6 : 7, then A : B : C : D is :

(a) 16 : 22 : 30 : 35 (b) 16 : 24 : 15 : 35

(c) 16 : 24 : 30 : 35 (d) 18 : 24 : 30 : 35

7. If 2A = 3B = 4C, then A : B : C is :

(a) 2 : 3 : 4 (b) 4 : 3 : 2

(c) 6 : 4 : 3 (d) 20 : 15 : 2

8. If $\frac{A}{3}:\frac{B}{4}:\frac{C}{5}$, then A : B : C is :

(a) 4 : 3 : 5 (b) 5 : 4 : 3

(c) 3 : 4 : 5 (d) 20 : 15 : 2

9. If 2A = 3B and 4B = 5C, then A : C is

(a) 4 : 3 (b) 8 : 15 (c) 15 : 8 (d) 3 : 4

10. The ratio of $4^{3.5} : 2^5$ is same as :

(a) 2 : 1 (b) 4 : 1 (c) 7 : 5 (d) 7 : 10

11. If $\frac{1}{5}:\frac{1}{x}=\frac{1}{x}:\frac{1}{125}$, then the value of x is :

(a) 1.5 (b) 2 (c) 2.5 (d) 3.5

12. If 0.75 : x : : 5 : 8, then x is equal to

(a) 1.12 (b) 1.20 (c) 1.25 (d) 1.30

13. If x : y = 5 : 2, then (8x + 9y) : (8x + 2y) is :

(a) 22 : 29 (b) 26 : 61

(c) 29 : 22 (d) 61 : 26

14. If 15% of x = 20 % of y, then x : y is

(a) 3 : 4 (b) 4 : 3

(c) 17 : 16 (d) 16 : 17

15. If (x : y) = 2 : 1, then $(x^2 - y^2) : (x^2 + y^2)$ is :

(a) 3 : 5 (b) 5 : 3 (c) 1 : 3 (d) 3 : 1

16. If $(4x^2 - 3y^2) : (2x^2 + 5y^2) = 12 : 19$, then (x : y) is :

(a) 2 : 3 (b) 1 : 2 (c) 3 : 2 (d) 2 : 1

17. If $x^2 + 4y^2 = 4xy$, then x : y is

(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 1 : 4

18. If $5x^2 - 13xy + 6y^2 = 0$, then x : y is

(a) (2 : 1) only (b) (3 : 5) only

(c) (5 : 3) or (1 : 2) (d) (3 : 5) or (2 : 1)

19. If $\frac{x}{5} = \frac{y}{8}$, then (x + 5) : (y + 8) is equal to :

(a) 3 : 5 (b) 13 : 8 (c) 8 : 5 (d) 5 : 8

20. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ is equal to :

(a) 7 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{7}$

HINT'S & SOLUTION

Sol.1 A : B = 5 : 7, B : C = 6 : 11 = $\left(6\times\frac{7}{6}\right) : \left(11\times\frac{7}{6}\right) = 7 : \frac{77}{6}$.

$\therefore$ A : B : C = $5 : 7 : \frac{77}{6}$ = 30 : 42 : 77.

Sol.2 $\left(\frac{A}{B} = \frac{3}{4}, \frac{B}{C} = \frac{8}{9}\right) \Rightarrow \frac{A}{C} = \left(\frac{A}{B}\times\frac{B}{C}\right)$

$= \left(\frac{3}{4}\times\frac{8}{9}\right) = \frac{2}{3} \Rightarrow A : C = 2 : 3.$

Sol.3 $\frac{A}{B} = \frac{8}{15}, \frac{B}{C} = \frac{5}{8}$ and $\frac{C}{D} = \frac{4}{5}$

$\Rightarrow \frac{A}{D} = \left(\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}\right) = \left(\frac{8}{15}\times\frac{5}{8}\times\frac{4}{5}\right) = \frac{4}{15}$

$\Rightarrow$ A : D = 4 : 15.

Sol.4 Let A = 2x, B = 3x and C = 4x. Then, $\frac{A}{B} = \frac{2x}{3x} = \frac{2}{3}, \frac{B}{C} = \frac{3x}{4x} = \frac{3}{4}$ and $\frac{C}{A} = \frac{4x}{2x} = \frac{2}{1}$.

$\Rightarrow \frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{2}{1} = 8 : 9 : 24.$

Sol.5 A : B = $\frac{1}{2} : \frac{3}{8}$ = 4 : 3, B : C = $\frac{1}{3} : \frac{5}{9}$ = 3 : 5,

C : D = $\frac{5}{6} : \frac{3}{4}$ = 10 : 9.

$\Rightarrow$ A : B = 4 : 3, B : C = 3 : 5 and C : D = $5 : \frac{9}{2}$

$\Rightarrow$ A : B : C : D = $4 : 3 : 5 : \frac{9}{2}$

= 8 : 6 : 10 : 9

Sol.6 A : B = 2 : 3, B : C = 4 : 5 = $\left(4\times\frac{3}{4}\right) : \left(5\times\frac{3}{4}\right) = 3 : \frac{15}{4}$ and C : D = 6 : 7

$= \left(6\times\frac{15}{24}\right) : \left(7\times\frac{15}{24}\right) = \frac{15}{4} : \frac{35}{8}$

$\Rightarrow$ A : B : C : D = $2 : 3 : \frac{15}{4} : \frac{35}{8}$

= 16 : 24 : 30 : 35.

Sol.7 Let 2A = 3B = 4C = k. Then, A = $\frac{k}{2}$, b = $\frac{k}{3}$, c = $\frac{k}{4}$.

$\Rightarrow$ A : B : C = $\frac{k}{2} : \frac{k}{3} : \frac{k}{4}$ = 6 : 4 : 3.

Sol.8 Let $\frac{A}{3} = \frac{B}{4} = \frac{C}{5} = k$. Then, A = 3*k*, B = 4*k* and C = 5k.

$\Rightarrow$ A : B : C = 3*k* : 4*k* : 5*k* = 3 : 4 : 5.

Sol.9 2A = 3B and 4B = 5C $\Rightarrow$ and $\frac{A}{B} = \frac{3}{2}$ and $\frac{B}{C} = \frac{5}{4}$

$\Rightarrow \frac{A}{C} = \left(\frac{A}{B}\times\frac{B}{C}\right) = \left(\frac{3}{2}\times\frac{5}{4}\right) = \frac{15}{8}$

= A : C = 15 : 8.

Sol.10 $\frac{4^{3.5}}{2^5} = \frac{(2^2)^{3.5}}{2^5} = \frac{2^{(2\times3.5)}}{2^5} = \frac{2^7}{2^5} = 2^2 = 4.$

$\therefore$ Required ratio is 4 : 1.

Sol.11 $\frac{1}{5} : \frac{1}{x} = \frac{1}{x} : \frac{100}{125} \Rightarrow \frac{1}{x}\times\frac{1}{x} = \left(\frac{1}{5}\times\frac{100}{125}\right) = \frac{4}{25}$

$\Rightarrow \frac{1}{x^2} = \frac{4}{25} \Rightarrow x^2 = \frac{25}{4} \Rightarrow x = \frac{5}{2} = 2.5.$

Sol.12 $(x \times 5) = (0.75 \times 8) \Rightarrow x = \frac{6}{5} = 1.20.$

Sol.13 Let x = 5*k* and *y* = 2*k*. Then, $\frac{8x+9y}{8x+2y}$

$= \frac{(8\times 5k)+(9\times 2k)}{(8\times 5k)+(2\times 2k)} = \frac{58k}{44k} = \frac{29}{22}.$

$\Rightarrow (8x + 9y) : (8x + 2y) = 29 : 22.$

Sol.14 15% of x = 20% of *y* $\Rightarrow \frac{15x}{100} \Rightarrow \frac{20y}{100}$

$= \frac{x}{y} = \left(\frac{20}{100}\times\frac{100}{15}\right) = \frac{4}{3}$

$\Rightarrow x : y = 4 : 3.$

Sol.15 $\frac{x}{y} = \frac{2}{1} \Leftrightarrow \frac{x^2}{y^2} = \frac{4}{1}$

$\Leftrightarrow \frac{x^2+y^2}{x^2-y^2} = \frac{4+1}{4-1}.$

[By componendo and dividendo]

$\Leftrightarrow \frac{x^2-y^2}{x^2+y^2} = \frac{3}{5} \Leftrightarrow (x^2 - y^2) : (x^2 + y^2)$

$= 3 : 5.$

Sol.16 $\frac{4x^2-3y^2}{2x^2+5y^2} = \frac{12}{19} \Leftrightarrow 19(4x^2 - 3y^2)$

$= 12(2x^2 + 5y^2)$

$\Leftrightarrow 52x^2 = 117y^2 \Leftrightarrow 4x^2 = 9y^2 \Leftrightarrow \frac{x^2}{y^2} = \frac{9}{4}$

$\Leftrightarrow \frac{x}{y} = \frac{3}{2}.$

$\therefore$ Required ratio is 3 : 2.

Sol.17 $x^2 + 4y^2 = 4xy \Leftrightarrow x^2 - 4xy + 4y^2 = 0$

$\Leftrightarrow (x - 2y)^2 = 0$

$\Leftrightarrow (x - 2y) = 0 \Leftrightarrow x = 2y \Leftrightarrow \frac{x}{y} = \frac{2}{1}.$

Sol.18 $5x^2 - 13xy + 6y^2 = 0$

$\Leftrightarrow 5x^2 - 10xy - 3xy + 6y^2 = 0$

$\Leftrightarrow 5x(x - 2y) == 3y(x - 2y) = 0$

$\Leftrightarrow (x - 2y)(5x = 3y) = 0$

$\Leftrightarrow x = 2y$ or $5x = 3y$

$\Leftrightarrow \frac{x}{y} = \frac{2}{1}$ or $\frac{x}{y} = \frac{3}{5}.$

$\therefore (x : y) = (2 : 1)$ or $(3 : 5).$

Sol.19 Let $\frac{x}{5} = \frac{y}{8} = k$. Then, x = 5*k* and *y* = 8k.

$\therefore \frac{x+5}{y+8} = \frac{5k+5}{8k+8} = \frac{5(k+1)}{8(k+1)} = \frac{5}{8}$

$\Rightarrow (x + 5) : (y + 8) = 5 : 8.$

Sol.20 Let $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$. Then, x = 5*k* and *y* = 8k.

$\therefore \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = \frac{14k}{7k} = 2.$