1 – ALGEBRA

INTRODUCTION

We have already come across simple algebraic expressions like x + 3, y – 5, 4x + 5, 10y – 5 and so on. In Class VI, we have seen how these expressions are useful in formulating puzzles and problems. We have also seen examples of several expressions in the chapter on simple equations.

Expressions are a central concept in algebra. This Chapter is devoted to algebraic expressions. When you have studied this Chapter, you will know how algebraic expressions are formed, how they can be combined, how we can find their values and how they can be used.

HOW ARE EXPRESSIONS FORMED?

We now know very well what a variable is. We use letters x, y, l, m, ... etc. to denote variables. A **variable** can take various values. Its value is not fixed. On the other hand, a **constant** has a fixed value. Examples of constants are: 4, 100, –17, etc.

We combine variables and constants to make algebraic expressions. For this, we use the operations of addition, subtraction, multiplication and division. We have already come across expressions like 4x + 5, 10y – 20. The expression 4x + 5 is obtained from the variable x, first by multiplying x by the constant 4 and then adding the constant 5 to the product. Similarly,

10y – 20 is obtained by first multiplying y by 10 and then subtracting 20 from the product.

The above expressions were obtained by combining variables with constants. We can also obtain expressions by combining variables with themselves or with other variables.

Look at how the following expressions are obtained:

$$x^2,\ 2y^2,\ 3x^2 - 5,\ xy,\ 4xy + 7$$

(i) The expression x^2 is obtained by multiplying the variable x by itself;

$$x \times x = x^2$$

Just as 4×4 is written as 4^2, we write $x \times x = x^2$. It is commonly read as x squared.

(Later, when you study the chapter 'Exponents and Powers' you will realise that x^2 may also be read as x raised to the power 2).

In the same manner, we can write $x \times x \times x = x^3$

Commonly, x^3 is read as 'x cubed'. Later, you will realise that x3 may also be read as x raised to the power 3.

x, x^2, x^3, ... are all algebraic expressions obtained from x.

(ii) The expression $2y^2$ is obtained from y: $2y^2 = 2 \times y \times y$

Here by multiplying y with y we obtain y^2 and then we multiply y^2 by the constant 2.

(iii) In $(3x^2 - 5)$ we first obtain x^2, and multiply it by 3 to get $3x^2$.

From $3x^2$, we subtract 5 to finally arrive at $3x^2 - 5$.

(iv) In xy, we multiply the variable x with another variable y. Thus, $x \times y = xy$.

(v) In $4xy + 7$, we first obtain xy, multiply it by 4 to get 4xy and add 7 to 4xy to get the expression.

TERMS OF AN EXPRESSION

We shall now put in a systematic form what we have learnt above about how expressions are formed. For this purpose, we need to understand what **terms** of an expression and their **factors** are.

Consider the expression $(4x + 5)$. In forming this expression, we first formed 4x separately as a product of 4 and x and then added 5 to it. Similarly consider the expression $(3x^2 + 7y)$. Here we first formed $3x^2$ separately as a product of 3, x and x. We then formed 7y separately as a product of 7 and y. Having formed $3x^2$ and 7y separately, we added them to get the expression.

You will find that the expressions we deal with can always be seen this way. They have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are known as **terms**. Look at the expression $(4x^2 – 3xy)$. We say that it has two terms, $4x^2$ and $–3xy$. The term $4x^2$ is a product of 4, x and x, and the term $(–3xy)$ is a product of $(–3)$, x and y.

Terms are added to form expressions. Just as the terms 4x and 5 are added to form the expression $(4x + 5)$, the terms $4x^2$ and $(–3xy)$ are added to give the expression

$(4x^2 – 3xy)$. This is because $4x^2 + (–3xy) = 4x^2 – 3xy$.

Note, the minus sign (–) is included in the term. In the expression $4x^2 –3xy$, we took the term as $(–3xy)$ and not as $(3xy)$. That is why we do not need to say that terms are 'added or subtracted' to form an expression; just 'added' is enough.

FACTORS OF A TERM

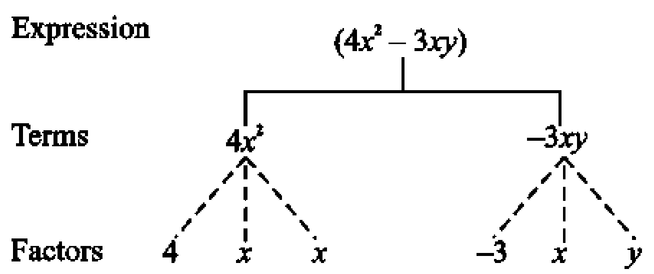
We saw above that the expression $(4x^2 – 3xy)$ consists of two terms $4x^2$ and $–3xy$. The term $4x^2$ is a product of 4, x and x; we say that 4, x and x are the factors of the term $4x^2$.

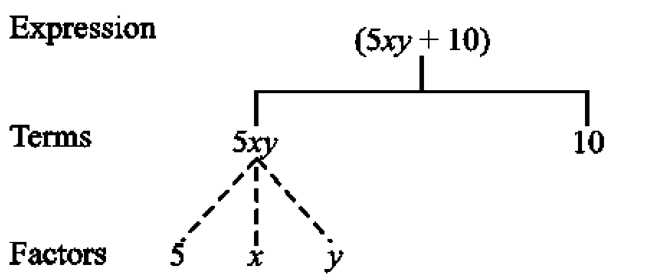
A term is a product of its factors. The term $–3xy$ is a product of the factors –3, x and y.

We can represent the terms and factors of the terms of an expression conveniently and elegantly by a tree diagram. The tree for the expression $(4x^2 – 3xy)$ is as shown in the adjacent figure.

Note, in the tree diagram, we have used dotted lines for factors and continuous lines for terms. This is to avoid mixing them.

Let us draw a tree diagram for the expression $5xy + 10$.





The factors are such that they cannot be further factorised. Thus we do not write 5xy as $5 \times xy$, because xy can be further factorised.

Similarly, if x^3 were a term, it would be written as $x \times x \times x$ and not $x^2 \times x$. Also, remember that 1 is not taken as a separate factor.

COEFFICIENTS

We have learnt how to write a term as a product of factors.

One of these factors may be numerical and the others algebraic (i.e., they contain variables). The numerical factor is said to be the numerical coefficient or simply the coefficient of the term.

It is also said to be the coefficient of the rest of the term (which is obviously the product of algebraic factors of the term). Thus in 5xy, 5 is the coefficient of the term. It is also the coefficient of xy. In the term 10xyz, 10 is the coefficient of xyz, in the term $-7x^2y^2$, -7 is the coefficient of x^2y^2.

When the coefficient of a term is +1, it is usually omitted.

For example, 1x is written as x; $1\,x^2y^2$ is written as x^2y^2 and so on. Also, the coefficient (-1) is indicated only by the minus sign. Thus $(-1)\,x$ is written as $-x$; $(-1)\,x^2\,y^2$ is written as $-x^2\,y^2$ and so on. Sometimes, the word 'coefficient' is used in a more general way. Thus we say that in the term 5xy, 5 is the coefficient of xy, x is the coefficient of $5y$ and y is the coefficient of 5x. In $10xy^2$, 10 is the coefficient of xy^2, x is the coefficient of 10y2 and y2 is the coefficient of 10x. Thus, in this more general way, a coefficient may be either a numerical factor or an algebraic factor or a product of two or more factors. It is said to be the coefficient of the product of the remaining factors.

Ex.1 Identify, in the following expressions, terms which are not constants. Give their numerical coefficients:

$xy + 4$, $13 - y^2$, $13 - y + 5y^2$, $4p^2q - 3pq^2 + 5$

Sol.

S. No.	Expression	Term (Which is not Constant)	Numerical Coefficient
(i)	$xy + 4$	xy	1
(ii)	$13 - y^2$	$-y^2$	-1
(iii)	$13 - y + 5y^2$	$-y$ $5y^2$	-1 5
(iv)	$4p^2\,q - 3pq^2 + 5$	$4p^2\,q -$ $3pq^2$	4 -3

Ex.2

(a) What are the coefficients of x in the following expressions?

$4x - 3y$, $8 - x + y$, $y^2x - y$, $2z - 5xz$

(b) What are the coefficients of y in the following expressions?

$4x - 3y$, $8 + yz$, $yz^2 + 5$, $my + m$

Sol.

(a) In each expression we look for a term with x as a factor. The remaining part of that term is the coefficient of x.

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	$4x - 3y$	$4x$	4
(ii)	$8 - x + y$	$-x$	-1
(iii)	$y^2x - y$	y^2x	y^2
(iv)	$2z - 5xz$	$-5xz$	$-5z$

(b) The method is similar to that in (a) above.

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	4x – 3y	– 3y	– 3
(ii)	8 + yz	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	my + m	my	m

LIKE AND UNLIKE TERMS

When terms have the same algebraic factors, they are **like** terms. When terms have different algebraic factors, they are **unlike** terms. For example, in the expression 2xy – 3x + 5xy – 4, look at the terms 2xy and 5xy. The factors of 2xy are 2, x and y. The factors of 5xy are 5, x and y. Thus their algebraic (i.e., those which contain variables) factors are the same and hence they are like terms. On the other hand the terms 2xy and –3x, have different algebraic factors.

They are unlike terms. Similarly, the terms, 2xy and 4, are unlike terms. Also, the terms –3x and 4 are unlike terms.

USING PRONUMERALS

In algebra we are doing arithmetic with just one new feature – we use letters to represent numbers. Because the letters are simply stand-ins for numbers, arithmetic is carried out exactly as it is with numbers. In particular the laws of arithmetic (commutative, associative and distributive) hold.

For example, the identities

2 + x = x + 2 2 × x = x × 2

(2 + x) + y = 2 + (x + y) (2 × x) × y = 2 × (x × y)

6(3x + 1) = 18x + 6

hold when x and y are any numbers at all.

In this module we will use the word **pronumeral** for the letters used in algebra. We choose to use this word in school mathematics because of confusion that can arise from the words such as 'variable'. For example, in the formula $E = mc^2$, the pronumerals E and m are variables whereas c is a constant.

Pronumerals are used in many different ways. For example:

- **Substitution:** 'Find the value of 2x + 3 if x = 4.' In this case the pronumeral is given the value 4.
- **Solving an equation:** 'Find x if 2x + 3 = 8.' Here we are seeking the value of the pronumeral that makes the sentence true.
- **Identity:** 'The statement of the commutative law: a + b = b + a.' Here a and b can be any real numbers.
- **Formula:** 'The area of a rectangle is A = lw.' Here the values of the pronumerals are connected by the formula.
- **Equation of a line or curve:** 'The general equation of the straight line is y = mx + c.' Here m and c are parameters. That is, for a particular straight line, m and c are fixed.

In some languages other than English, one distinguishes between 'variables' in functions and 'unknown quantities' in equations ('incógnita' in Portuguese/Spanish, 'inconnue' in French) but this does not completely clarify the situation. The terms such as variable and parameter cannot be precisely defined at this stage and are best left to be introduced later in the development of algebra.

An **algebraic expression** is an expression involving numbers, parentheses, operation signs and pronumerals that becomes a number when numbers are substituted for the pronumerals. For example $2x + 5$ is an expression but +) × is not.

Examples of algebraic expressions are:

$$3x + 1 \text{ and } 5(x^2 + 3x)$$

As discussed later in this module the multiplication sign is omitted between letters and between a number and a letter. Thus substituting $x = 2$ gives

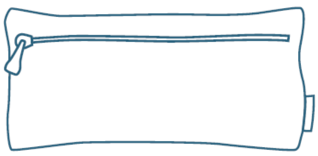
$3x + 1 = 3 \times 2 + 1 = 7$ and $5(x^2 + 3x) = 5(2^2 + 3 \times 2) = 30$.

In this module, the emphasis is on expressions, and on the connection to the arithmetic that students have already met with whole numbers and fractions. The values of the pronumerals will therefore be restricted to the whole numbers and non-negative fractions.

CHANGING WORDS TO ALGEBRA

In algebra, pronumerals are used to stand for numbers. For example, if a box contains x stones and you put in five more stones, then there are $x + 5$ stones in the box. You may or may not know what the value of x is (although in this example we do know that x is a whole number).

- Joe has a pencil case that contains an unknown number of pencils. He has three other pencils. Let x be the number of pencils in the pencil case. Then Joe has $x + 3$ pencils altogether.



- Theresa has a box with least 5 pencils in it, and 5 are removed. We do not know how many pencils there are in the pencil case, so let z be the number of pencils in the box.Then there are $z – 5$ pencils left in the box.
- There are three boxes, each containing the same number of marbles. If there are x marbles in each box, then the total number of marbles is $3 \times x = 3x$.

- If n oranges are to be divided amongst 5 people, then each person receives $\frac{n}{5}$ oranges.

(Here we assume that n is a whole number. If each person is to get a whole number of oranges, then n must be a multiple of 5.)

The following table gives us some meanings of some commonly occurring algebraic expressions.

$x + 3$	• The sum of x and 3 • 3 added to x, or x is added to 3 • 3 more than x, or x more than 3
$x – 3$	• The difference of x and 3 (where $x \geq 3$) • 3 is subtracted from x • 3 less than x • x minus 3

$x \div 3$	•	x divided by 3
	•	the quotient when x is divided by 3
$2 \times x - 3$	•	x is first multiplied by 2, then 3 is subtracted
$x \div 3 + 2$	•	x is first divided by 3, then 2 is added

NOTATIONS AND LAWS

Expressions with zeroes and ones

Zeroes and ones can often be eliminated entirely. For example:

$x + 0 = x$ (Adding zero does not change the number.)

$1 \times x = x$ (Multiplying by one does not change the number.)

ALGEBRAIC NOTATION

In algebra there are conventional ways of writing multiplication, division and indices.

(1) Notation for multiplication

The $\times$ sign between two pronumerals or between a pronumeral and a number is usually omitted. For example, $3 \times x$ is written as 3x and $a \times b$ is written as ab. We have been following this convention earlier in this module.

It is conventional to write the number first. That is, the expression $3 \times a$ is written as 3a and not as a3.

(2) Notation for division

The division sign $\div$ is rarely used in algebra. Instead the alternative fraction notation for division is used. We recall $24 \div 6$ can be written as $\frac{24}{6}$.

Using this notation, x divided by 5 is written as $\frac{x}{5}$, not as $x \div 5$.

(3) Index notation

$x \times x$ is written as x^2 and $y \times y \times y$ is written as y^3.

Ex.1 Write each of the following using concise algebraic notation.

(a) A number x is multiplied by itself and then doubled.

(b) A number x is squared and then multiplied by the square of a second number y.

(c) A number x is multiplied by a number y and the result is squared.

Sol. (a) $x \times x \times 2 = x^2 \times 2 = 2x^2$.

(b) $x^2 \times y^2 = x^2y^2$.

(c) $(x \times y)^2 = (xy)^2$ which is equal to x^2y^2.

Summary

- $2 \times x$ is written as 2x
- $x \times y$ is written as xy or yx
- $x \times y \times z$ is written as xyz
- $x \times x$ is written as x^2
- $4 \times x \times x \times x = 4x3$
- $x \div 3$ is written as $\frac{x}{3}$
- $x \div (yz)$ is written as $\frac{x}{yz}$
- $x^1 = x$ (the first power of x is x)
- $x^o = 1$
- $1x = x$
- $0x = 0$

SUBSTITUTION

Assigning values to a pronumeral is called **substitution**.

Ex.2 If $x = 4$, what is the value of:

(a) $5x$ (b) $x + 3$

(c) $x - 1$ (d) $\frac{x}{2}$

Sol. (a) $5x = 5 \times 4 = 20$ (b) $x + 3 = 4 + 3 = 7$

(c) $x - 1 = 4 - 1 = 3$ (d) $\frac{x}{2} = \frac{4}{2} = 2$

Ex.3 If $x = 6$, what is the value of:

(a) $3x + 4$ (b) $2x + 3$

(c) $2x - 5$ (d) $\frac{x}{2} - 2$

(e) $\frac{x}{3} + 2$

Sol. (a) $3x + 4 = 3 \times 6 + 4 = 22$

(b) $2x + 3 = 2 \times 6 + 3 = 15$

(c) $2x - 5 = 2 \times 6 - 5 = 7$

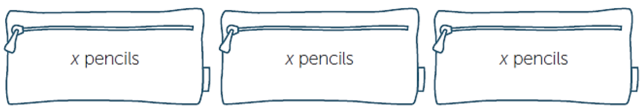
(d) $\frac{x}{2} - 2 = \frac{6}{2} - 2 = 1$

(e) $\frac{x}{3} + 2 = \frac{6}{3} + 2 = 4$

ADDING AND SUBTRACTING LIKE TERMS

Like terms

If you have 3 pencil case with the same number x of pencils in each, you have 3x pencils altogether.



If there are 2 more pencil cases with x pencils in each, then you have $3x + 2x = 5x$ pencils altogether. This can be done as the number of pencils in each case is the same. The terms 3x and 2x are said to be **like terms**.

Consider another example. If Jane has x packets of chocolates each containing y chocolates, then she has $x \times y = xy$ chocolates.

If David has twice as many chocolates as Jane, he has $2 \times xy = 2xy$ chocolates.

Together they have $2xy + xy = 3xy$ chocolates.

The terms 2xy and xy are **like terms**. Two terms are called **like terms** if they involve exactly the same pronumerals and each pronumeral has the same index.

Ex.4 Which of the following pairs consists of like terms:

(a) $3x, 5x$

(b) $4x^2, 8x$

(c) $4x^2y, 12x^2y$

Sol. (a) 5x and 3x are like terms.

(b) $4x^2$ and 8x are not like terms since the powers of x are different.

(c) $4x^2y, 12x^2y$ are like terms

ADDING AND SUBTRACTING LIKE TERMS

The distributive law explains the addition and subtraction of like terms. For example:

$2xy + xy = 2 \times xy + 1 \times xy = (2 + 1)xy = 3xy$

The terms 2x and 3y are not like terms because the pronumerals are different. The terms

3x and $3x^2$ are not like terms because the indices are different. For the sum $6x + 2y + 3x$,

the terms 6x and 3x are like terms and can be added. There are no like terms for 2y, so by

using the commutative law for addition the sum is

$6x + 2y + 3x = 6x + 3x + 2y = 9x + 2y$.

The **any-order principle for addition** is used for the adding like terms.

Because of the commutative law and the associative law for multiplication (any-order principle for multiplication) the order of the factors in each term does not matter.

$$5x \times 3y = 15xy = 15yx$$

$$6ab \times 3b^2a = 18a^2b^3 = 18b^3a^2$$

Like terms can be added and subtracted as shown in the example below.

Ex.5 Simplify each of the following by adding or subtracting like terms:

(a) $2x + 3x + 5x$

(b) $3xy + 2xy$

(c) $4x^2 – 3x^2$

(d) $2x^2 + 3x + 4x$

(e) $4x^2y – 3x^2y + 3xy^2$

Sol. (a) $2x + 3x + 5x = 10x$

(b) $3xy + 2xy = 5xy$

(c) $4x^2 – 3x^2 = x2$

(d) $2x^2 + 3x + 4x = 2x^2 + 7x$

(e) $4x^2y – 3x^2y + 3xy^2 = x^2y + 3xy^2$

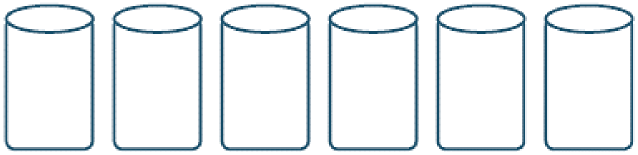
THE USE OF BRACKETS

Brackets fulfill the same role in algebra as they do in arithmetic. Brackets are used in algebra in the following way.

'Six is added to a number and the result is multiplied by 3.'

Let x be the number. Then the expression is $(x + 6) \times 3$. We now follow the convention that the number is written at the front of the expression $(x + 6)$ without a multiplication sign. The preferred form is thus $3(x + 6)$.

Ex.6 For a party, the host prepared 6 tins of chocolate balls, each containing n chocolate balls.



(a) The host places two more chocolates in each tin. How many chocolates are there altogether in the tins now?

(b) If n = 12, that is, if there were originally 12 chocolates in each box, how many chocolates are there altogether in the tins now?

Sol.

(a) The number of chocolates in each tin is now $n + 2$. There are 6 tins, and therefore there are $6 (n + 2)$ chocolates in total.

(b) If $n = 12$, the total number of chocolates is $6 (n + 2) = 6 (12 + 2) = 6 \times 14 = 84$.

USE OF BRACKETS AND POWERS

The following example shows the importance of following the conventions of order of operations when working with powers and brackets.

- $2x^2$ means $2 \times x^2$
- $(2x)^2 = 2x \times 2x = 4x^2$

Ex.7 For x = 3, evaluate each of the following:

(a) $2x^2$

(b) $(2x)^2$

Sol. (a) $2x^2 = 2 \times x \times x = 2 \times 3 \times 3 = 18$

(b) $(2x)^2 = (2 \times x)^2 = (2 \times 3)2 = 36$

MULTIPLYING TERMS

Multiplying algebraic terms involves the **any-order property** of multiplication discussed for whole numbers.

The following shows how the any-order property can be used

$3x \times 2y \times 2xy = 3 \times x \times 2 \times y \times 2 \times x \times y$

$= 3 \times 2 \times 2 \times x \times x \times y \times y$

$= 12x^2y^2$

Ex.7 Simplify each of the following:

(a) $5 \times 2a$ (b) $3a \times 2a$ (c) $5xy \times 2xy$

Sol. (a) $5 \times 2a = 10a$

(b) $3a \times 2a = 3 \times a \times 2 \times a = 6a^2$

(c) $5xy \times 2xy = 5 \times x \times y \times 2 \times x \times y = 10x^2y^2$

With practice no middle steps are necessary.

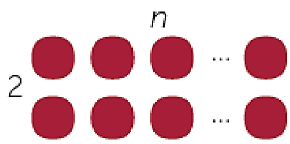
For example, $2a \times 3a \times 2a^2 = 12a^4$

ARRAYS AND AREAS

ARRAYS

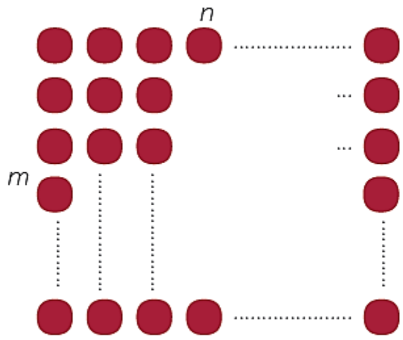
Arrays of dots have been used to represent products in the module Multiplication of Whole Numbers.

For example $2 \times 6 = 12$ can be represented by 2 rows of 6 dots. The diagram below represents two rows with some number of dots.

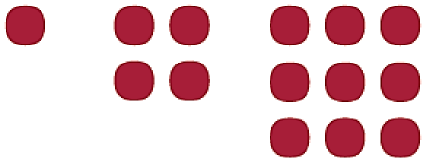


Let n be the number of dots in each row. Then there are $2 \times n = 2n$ dots.

If an array is m dots by n dots then there are mn dots. If the array is $m \times n$ then by convention we have m rows and n columns. We can represent the product $m \times n = mn$ by such an array.



Ex.8 The diagram shows squares formed by dots. The pattern goes on forever. How many dots are there in the n^{th} diagram?



Sol. In the 1^{st} diagram there are $1 \times 1 = 1^2$ dots.

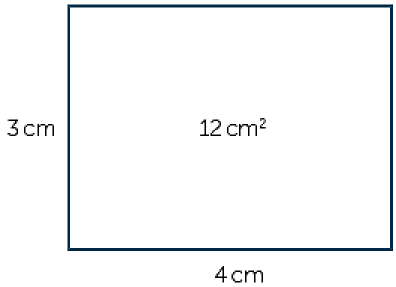
In the 2^{nd} diagram there are $2 \times 2 = 2^2$ dots.

In the 3^{rd} diagram there are $3 \times 3 = 3^2$ dots.

In the nth diagram there will be $n \times n = n^2$ dots.

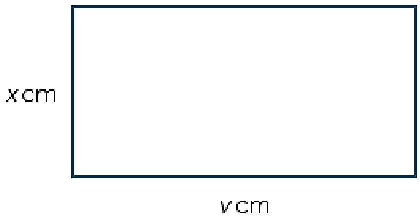
AREAS

We can also represent a product like 3×4 by a rectangle.



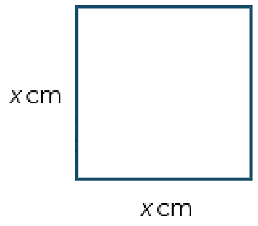
The area of a 3 cm by 4 cm rectangle is 12 cm^2.

The area of a x cm by y cm rectangle is $x \times y = xy$ cm^2.



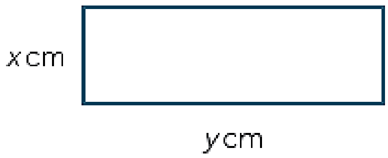
(x and y can be any positive numbers.)

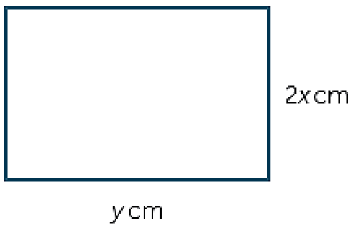
The area of a x cm by x cm square is x^2 cm^2.



(x can be any positive number.)

Ex.9 Find the total area of the two rectangles in terms of x and y.





Sol. The area of the rectangle to the left is xy cm^2 and the area of the rectangle to the right is 2xy cm^2. Hence the total area is xy + 2xy = 3xy cm^2.

NUMBER PATTERNS

Some simple statements with numbers demonstrate the convenience of algebra.

Ex.10 The nth positive even number is 2n.

(a) What is the square of the nth positive even number?

(b) If the nth positive even number is doubled what is the result?

Sol. (a) The square of the nth positive even number is $(2n)^2 = 4n^2$

(b) The double of the nth positive even number is $2 \times 2n = 4n$.

ALGEBRAIC FRACTIONS

Quotients of expressions involving pronumerals often occur. We call them **algebraic fractions** we will meet this again in the module, Special Expansions and Algebraic Fractions.

Ex.11 Write each of the following in algebraic notation.

(a) A number is divided by 5, and 6 is added to the result.

(b) Five is added to a number, and the result is divided by 3.

Sol. (a) Let x be the number.

Dividing by 5 gives $\frac{x}{5}$.

Adding 6 to this result gives $\frac{x}{5} + 6$.

(b) Let the number be x.

Adding 5 gives x + 5.

Dividing this by 3 gives $\frac{x+5}{3}$.

Notice that the vinculum acts as a bracket.

Ex.11 If x = 10, find the value :

(a) $\frac{x}{2}$ (b) $\frac{x}{5} + 3$

(c) $\frac{x+4}{3}$ (d) $\frac{x-4}{4}$

Sol. (a) $\frac{x}{2} = \frac{10}{2} = 5$

(b) $\frac{x}{5} + 3 = \frac{10}{5} + 3$

(c) $\frac{x+4}{3} = \frac{10+4}{3} = \frac{14}{3} = 4\frac{2}{3}$

(d) $\frac{x-4}{4} = \frac{10-4}{4} = \frac{6}{4} = \frac{3}{2}$

Ex.12 A vat contains n litres of oil. Forty litres of oil are then added to the vat.

(a) How many litres of oil are there now in the vat?

(b) The oil is divided into 50 containers. How much oil is there in each container?

Sol. (a) There is a total of n + 40 litres of oil in the vat.

(b) There are $\frac{n+40}{50}$ litres of oil in each container.

EXPANDING BRACKETS AND COLLECTING LIKE TERMS

Expanding brackets

Numbers obey the distributive laws for multiplication over addition and subtraction. For example:

$$3 \times (4 + 5) = 3 \times 4 + 3 \times 5$$

$$7 \times (6 - 3) = 7 \times 6 - 7 \times 3$$

The distributive laws for division over addition and subtraction also hold as shown.

For example:

$(8 + 6) \div 2 = 8 \div 2 + 6 \div 2$ and $\frac{9-7}{3} = \frac{9}{3} - \frac{7}{3}$

As with adding like terms and multiplying terms, the laws that apply to arithmetic can be extended to algebra. This process of rewriting an expression to remove brackets is usually referred to as **expanding brackets**.

Ex.13 Use the distributive law to rewrite these expressions without brackets.

(a) $5(x - 4)$

(b) $4(3x + 2)$

(c) $6(4 - 2x)$

Sol. (a) $5(x - 4) = 5x - 20$

(b) $4(3x + 2) = 12x + 8$

(c) $6(4 - 2x) = 24 - 12x$

Ex.14 Use the distributive law to rewrite these expressions without brackets.

(a) $\frac{3x+1}{3}$ (b) $\frac{2-x}{2}$

Sol. (a) $\frac{3x+1}{3} = x + \frac{1}{3}$

(b) $\frac{2-x}{2} = 1 - \frac{x}{2}$

Collecting like terms

After brackets have been expanded like terms can be collected.

Ex.15 Expand the brackets and collect like terms:

(a) $2(x - 6) + 5x$

(b) $3 + 3(x - 1)$

(c) $3(2x + 4) + 6(x - 1)$

Sol. (a) $2(x - 6) + 5x = 2x - 12 + 5x = 7x - 12$

(b) $3 + 3(x - 1) = 3 + 3x - 3 = 3x$

(c) $3(2x + 4) + 6(x - 1) = 6x + 12 + 6x - 6$

$= 12x + 6$

WORKSHEET

1. Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) a^2

(ii) $a^2 – b^2$

(iii) $x^3 + y^3 + z^3$

(iv) $x^3 + y^3 + z^3 + 3xyz$

(v) $7 + 5$

(vi) $a b c + 1$

(vii) $3x – 2 +5$

(viii) $2x – 3y + 4$

(ix) $x y + y z + z x$

(x) $ax^3 + bx^2 + cx + d$

2. Write all the terms of each of the following algebraic expressions:

(i) $3x$

(ii) $2x – 3$

(iii) $2x^2 – 7$

(iv) $2x^2 + y^2 – 3xy + 4$

3. Identify the terms and also mention the numerical coefficients of those terms:

(i) $4xy, – 5x^2y, –3yx, 2xy^2$

(ii) $7a^2bc$, $–3ca^2b$, $–(5/2)abc^2$, $3/2abc^2$, $(–4/3)cba^2$

4. Identify the like terms in the following algebraic expressions:

(i) $a^2 + b^2 – 2a^2 + c^2 +4a$

(ii) $3x + 4xy - + 52zy$

(iii) $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac — 2a^2bc + 3cab^2$.

5. Write the coefficient of x in the following:

(i) $– 12K$

(ii) $– 7xy$

(iii) xyz

(iv) $– 7ax$

6. Write the coefficient of x^2 in the following:

(i) $–3x^2$

(ii) $5x^2yz$

(iii) $5/7x^2z$

(iv) $(–3/2)ax^2 + yx$

7. Write the coefficient of:

(i) y in $– 3y$

(ii) a in $2ab$

(iii) z in $– 7xyz$

(iv) p in $– 3pqr$

(v) y^2 in $9xy^2z$

(vi) in $+1$

(vii) x^2 in $– x^2$

8. Write the numerical coefficient Of each in the following:

(i) xy

(ii) $– 6yz$

(iii) $7abc$

(iv) $– 2x^3y^2z$

9. Write the numerical coefficient of each term in the following algebraic expressions:

(i) $4x^2 y – (3/2)xy + 5/2xy^2$

(ii) $(–5/3)x^2y + (7/4)xyz + 3$

10. Write the constant term of each of the following algebraic expressions:

(i) $x^2y - xy^2 + 7xy^{-3}$

(ii) $a^3 - 3a^2 + 7a + 5$

11. Evaluate each of the following expressions for x

(i) $(x/y) + (y/z) + (z/x)$

(ii) $x^2 + + z^2 - xy - yz - zx$

12. Evaluate each of the following algebraic expressions for $x = 1$, $y = -1$, $z = 2$, $a = -2$, b

(i) $ax + by + cz$

(ii) $ax^2 + by^2 - cz$

(iii) $axy + byz + cxy$

14. Identify the like expressions.

5x, – 1 4x, + 1, xy, – 3xy

15. Subtract: $(4x + 5)$ from $(-3x + 7)$

16. Subtract: $3x^2 - 5x + 7$ from $5x^2 - 7x + 9$

17. Multiply $(6x^2 - 5x + 3)(3x^2 + 7x - 3)$

18. Simplify: $2x^2(x + 2) - 3x(x^2 - 3) - 5x(x + 5)$

19. Simplify the following:

(i) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$

(ii) $x^2(x - 3y^2) - xy(y^2 - 2xy) - x(y^3 - 5x^2)$

20. Multiply $(3x^2 + 5y^2)$ by $(5x^2 - 3y^2)$

HINT'S & SOLUTION

Sol.1

(i) Given a^2

a^2 is a monomial expression because it contains only one term

(ii) Given $a^2 - b^2$

$a^2 - b^2$ is a binomial expression because it contains two terms

(iii) Given $x^3 + y^3 + z^3$

$x^3 + y^3 + z^3$ is a trinomial because it contains three terms

(iv) Given $x^3 + y^3 + z^3 + 3xyz$

$x^3 + y^3 + z^3 + 3xyz$ is a quadrinomial expression because it contains four terms

(v) Given 7 + 5

7 + 5 is a monomial expression because it contains only one term

(vi) Given a b c + 1

a b c + 1 is a binomial expression because it contains two terms

(vii) Given 3x – 2 + 5

3x – 2 + 5 is a binomial expression because it contains two terms

(viii) Given 2x – 3y + 4

2x– 3y + 4 is a trinomial because it contains three terms

(ix) Given x y + y z + z x

x y + y z + z x is a trinomial because it contains three terms

(x) Given $ax^3 + bx^2 + cx + d$

$ax^3 + bx^2 + cx + d$ is a quadrinomial expression because it contains four terms

Sol.2

(i) Given 3x

3x is the only term of the given algebraic expression.

(ii) Given 2x – 3

2x and – 3 are the terms of the given algebraic expression.

(iii) Given $2x^2 - 7$

$2x^2$ and – 7 are the terms of the given algebraic expression.

(iv) Given $2x^2 + y^2 - 3xy + 4$

$2x^2$, y^2, – 3xy and 4 are the terms of the given algebraic expression.

Sol.3

(i) Like terms 4xy, –3yx and Numerical coefficients 4, –3

(ii) Like terms ($7a^2bc$, $-3ca^2b$) and ($-4/3cba^2$) and their Numerical coefficients 7, –3, (–4/3)

Like term are ($-5/2abc^2$) and ($3/2abc^2$) and numerical coefficient are (–5/2) and (3/2).

Sol.4

(i) Given $a^2 + b^2 - 2a^2 + c^2 + 4a$

The like terms in the given algebraic expressions are a^2 and $-2a^2$.

(ii) Given 3x + 4xy – 2yz + 52zy

The like terms in the given algebraic expressions are – 2yz and 52zy

(iii) Given $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac - 2a^2bc + 3cab^2$.

The like terms in the given algebraic expressions are ab^2c, $2acb^2$, b^2ac and $3cab^2$.

Sol.5

(i) Given $-12x$

The numerical coefficient of x is -12.

(ii) Given $-7xy$

The numerical coefficient of x is $-7y$.

(iii) Given xyz

The numerical coefficient of x is yz.

(iv) Given $-7ax$

The numerical coefficient of x is $-7a$.

Sol.6

(i) Given $-3x^2$

The numerical coefficient of x^2 is -3.

(ii) Given $5x^2yz$

The numerical coefficient Of x^2 is 5yz.

(iii) Given$5/7x^2z$

The numerical coefficient of x^2 is 5/7z.

(iv) Given $(-3/2)\ ax^2 + yx$

The numerical coefficient of x^2 is $-(3/2)$ a.

Sol.7

(i) Given $-3y$

The coefficient of y is -3.

(ii) Given 2ab

The coefficient of a is 2b.

(iii) Given $-7xyz$

The coefficient of z is $-7xy$.

(iv) Given $-3pqr$

The coefficient of p is $-3qr$.

(v) Given $9xy^2z$

The coefficient of y^2 is 9xz.

(vi) Given $x^3 + 1$

The coefficient of x^3 is 1.

(vii) Given $-x^2$

The coefficient of x^2 is -1.

Sol.8

(i) Given xy

The numerical coefficient in the term xy is 1.

(ii) Given $-6yz$

The numerical coefficient in the term $-6yz$ is -6.

(iii) Given 7abc

The numerical coefficient in the term 7abc is 7.

(iv) Given $-2x^3y^2z$

The numerical coefficient in the term $-2x^3y^2z$ is -2.

Sol.9

(i) Given $4x^2y - (3/2)\ xy + 5/2\ xyz$

Numercial coefficient of following algebraic expressions are given below:

Term	Coefficient
$4x^2\ y$	4
$-(3/2)xy$	$-(3/2)$
$5/2\ xy^2$	(5/2)

(ii) Given $(-5/3)x^2\ y + (7/4)xyz + 3$

Numerial coefficient of following algebraic expressions are given below:

Term	Coefficient
$(-5/3)x^2\ y$	$(-5/3)$
(7/4)xyz	(7/4)
3	3

Sol.10

(i) Given $x^2y - xy^2 + 7xy - 3$

The constant term in the given algebraic expressions is -3.

(ii) Given $a^3 - 3a^2 + 7a + 5$

The constant term in the given algebraic expressions is 5.

Sol.11

(i) Given $x = -2, y = -1, z = 3$

Consider $(x/y) + (y/z) + (z/x)$

On substituting the given values we get,

$= (-2/-1) + (-1/3) + (3/-2)$

The LCM of 3 and 2 is 6

$= (12 - 2 - 9)/6$

$= (1/6)$

(ii) Given $x = -2, y = -1, z = 3$

Consider $x^2 + y^2 + z^2 - xy - yz - zx$

On substituting the given values we get,

$= (-2)^2 + (-1)^2 + 3^2 - (-2)(-1) - (-1)(3) - (3)(-2)$

$= 4 + 1 + 9 - 2 + 3 + 6 = 23 - 2 = 21$

Sol.12

(i) Given $x = 1, y = -1, z = 2, a = -2, b = 1, c = -2$

Consider $ax + by + cz$

On substituting the given values

$= (-2)(1) + (1)(-1) + (-2)(2)$

$= -2 - 1 - 4$

$= -7$

(ii) Given $x = 1, y = -1, z = 2, a = -2$, b

Consider $ax^2 + by^2 - cz$

On substituting the given values

$= (-2) \times 1^2 + 1 \times (-1)^2 - (-2) \times 2$

$= -1 + 4 = 3$

(iii) Given $x = 1, y = -1, z = 2, a = -2, b = 1, c = -2$

Consider $axy + byz + cxy$

$= (-2) \times 1 \times -1 + 1 \times -1 \times 2 + (-2) \times 1 \times (-1)$

$= 2 + (-2) + 2$

$= 4 - 2$

$= 2$

Sol.13

(i) Monomials:

(a) $3x$

(b) $5xy^2$

(ii) Binomials:

(a) $p + q$

(b) $-5a + 2b$

(iii) Trinomials.

(a) $a + b + c$

(b) $x^2 + x = 2$

Sol.14 Like terms: $5x$ and $-1\,4x$, x^2 and $-9x^2$, xy and $-3xy$

Sol.15 $(-3x + 7) - (4x + 5)$

$= -3x + 7 - 4x - 5$

$= -3x - 4x + 7 - 5$

$= -7x + 2$

Sol.16 $(5x^2 - 7x + 9) - (3x^2 - 5x + 7)$

$= 5x^2 - 7x + 9 - 3x^2 + 5x - 7$

$= 5x^2 - 3x^2 + 5x - 7x + 9 - 7$

$= 2x^2 - 2x + 2$

Sol.17 $(6x^2 - 5x + 3)(3x^2 + 7x - 3)$

$= (6x^2 - 5x + 3) - 5x(3x^2 + 7x - 3) + 3(3x^2 + 7x - 3)$

$= 18x^4 + 42x^3 - 18x^2 - 15x^3 - 35x^2 + 15x + 9x^2 + 21x - 9$

$= 18x^4 + 27x^3 - 44x^2 + 36x - 9$

Sol.18 $2x^2(x + 2) - 3x(x^2 - 3) - 5x(x + 5)$

$= 2x^3 + 4x^2 - 3x^3 + 9x - 5x^2 - 25x$

$= 2x^3 - 3x^3 - 5x^2 + 4x^2 + 9x - 25x$

$= -x^3 - x^2 - 16x$

Sol.19

(i) $a^2(b^2 - c^2) + b^2 (c^2 - a^2) + c^2 (a^2 - b^2)$

$= a^2b^2 - a^2c^2) + b^2 c^2 - b^2 a^2) + c^2a^2 - c^2 b^2)$

$= 0$

(ii) $x^2(x - 3y^2) - xy(y^2 - 2xy) - x(y^3 - 5x^2)$

$= x^3 - 3x^2y^2 - xy3 + 2x^2y^2 - xy^3 + 5x^3$

$= x^3 + 5x^3 - 3x^2y^2 + 2x^2y^2 - xy^3 + xy^3$

$= 6x^3 - x^2y^2 - 2xy^3$

Sol.20 $(3x^2 + 5y^2)$ by $(5x^2 - 3y^2)$

$= 3x^2(5x^2 - 3y^2) + 5y^2(5x^2 - 3y^2)$

$= 15x^4 - 9x^2y^2 + 25x^2y^2 - 15y^4$

$= 15x^4 + 16x^2y^2 - 15y^4$