CHAPTER 1

Basic Mathematics & Logarithm

SECTION - A : BASIC MATHS

NUMBER SYSTEM

Natural Numbers

The counting numbers 1, 2, 3, 4 .... are called Natural Numbers. The set of natural numbers is denoted by N. Thus, N = {1, 2, 3, 4,......}. N is also denoted by I^+ or Z^+

Whole Numbers

Natural numbers including zero are called whole numbers. The set of whole numbers, is denoted by W. Thus W = {0, 1, 2,......}. W is also called as set of non-negative integers.

Integers

The numbers..– 3, – 2, –1, 0, 1, 2, ... are called integers and the set is denoted by I or Z.

Thus I (or Z) = {...–3, –2, –1, 0, 1, 2, 3.........}

1. Set of positive integers, denoted by I^+ and consists of {1, 2, 3, ...}
2. Set of negative integers, denoted by I^- and consists of {.........., –3, –2, –1}
3. Set of non-negative integers {0, 1, 2, 3,..........}
4. Set of non-positive integers {...., –3, –2, –1, 0}

Even Integers

Integers which are divisible by 2 are called even integers. e.g. 0, ± 2, ± 4,.....

Odd Integers

Integers which are not divisible by 2 are called as odd integers. e.g. ± 1, ± 3, ...

Prime Number

Let 'p' be a natural number, 'p' is said to be prime if it has exactly two distinct factors, namely 1 and itself.
e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,......

Remarks

1. '1' is neither prime nor composite.
2. '2' is the only even prime number.

Composite Number

Let 'a' be a natural number, 'a' is said to be composite if it
has atleast three distinct factors.

Co-prime Numbers

Two natural numbers (not necessarily prime) are coprime, if their H.C.F.(Highest common factor) is one.e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) etc.
These numbers are also called as **relatively prime** numbers.

Remarks

1. Numbers which are not prime are composite numbers (except 1)
2. '4' is the smallest composite number.
3. Two distinct prime numbers are always co-prime but converse need not be true.
4. Consecutive numbers are always co-prime numbers.

Twin Prime Numbers

If the difference between two prime numbers is two, then the numbers are called as twin prime numbers.
eg. {3, 5}, {5, 7}, {11, 13}, {17, 19}, {29, 31}

Rational Numbers

All the numbers those can be represented in the form p/q, where p and q are integers and q ≠ 0, are called rational numbers and their set is denoted by Q.

Thus, Q = $\{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\}$. It may be noted that every integer is a rational numbers.

Irrational Numbers

There are real numbers which cannot be expressed in p/q form. These numbers are Called irrational numbers and their set is denoted by Q^c or Q'.

(i.e. complementary set of Q) e.g. $\sqrt{2}$, $1+\sqrt{3}$, e, π etc. Irrational numbers can not be expressed as recurring decimals.

Remark :

1. e. ≈ 2.71 is called Napier's constant and $\pi \approx 3.14$.

SURDS

If a is not a perfect nth power, then $\sqrt[n]{a}$ is called a surd of the nth order.

In an expression of the form $\frac{a}{\sqrt{b}+\sqrt{c}}$, the denominator can be rationalized by multiplying numerator and the denominator by $\sqrt{b}-\sqrt{c}$ which is called the conjugate of $\sqrt{b}+\sqrt{c}$. If $x+\sqrt{y} = a+\sqrt{b}$ where x, y, a, b are rationals, then x = a and y = b.

Example 1

Prove that $\log_3 5$ is irrational.

Sol. Let $\log_3 5$ is rational.

$\therefore \log_3 5 = \frac{p}{q}$; where p and q are co-prime numbers.

$\Rightarrow 3^{p/q} = 5 \Rightarrow 3^p = 5^q$. which is not possible, hence our assumption is wrong and $\log_3 5$ is irrational.

Example 2

Simplify : $\frac{12}{3+\sqrt{5}-2\sqrt{2}}$

Sol. The expression =

$$\frac{12(3+\sqrt{5}+2\sqrt{2})}{(3+\sqrt{5})^2-(2\sqrt{2})^2} = \frac{12(3+\sqrt{5}+2\sqrt{2})}{6+6\sqrt{5}}$$

$$= \frac{2(3+\sqrt{5}+2\sqrt{2})(\sqrt{5}-1)}{(\sqrt{5}+1)\times(\sqrt{5}-1)}$$

$$= \frac{2(2+2\sqrt{5}+2\sqrt{10}-2\sqrt{2})}{4}$$

$$= 1+\sqrt{5}+\sqrt{10}-\sqrt{2}$$

Example 3

Find the square root of $7+2\sqrt{10}$

Sol. Let $\sqrt{7+2\sqrt{10}} = \sqrt{x}+\sqrt{y}$.

Squaring, $x+y+2\sqrt{xy} = 7+2\sqrt{10}$

Hence, x + y = 7 and xy = 10.These two relation give

x = 5, y = 2. Hence $\sqrt{7+2\sqrt{10}} = \sqrt{5}+\sqrt{2}$

Remark :

1. $\sqrt{\ }$ symbol stands for the positive square root only.

Example 4

Prove that $\sqrt[3]{2}$ cannot be represented in the form $p+\sqrt{q}$,

where p and q are rational (q > 0 and is not a perfect square).

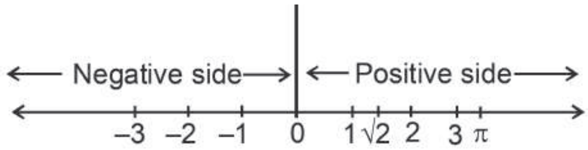
Sol. Put $\sqrt[3]{2} = p+\sqrt{q}$. Hence $2 = p^3+3pq+(3p^2+q)\sqrt{q}$, Since q is not a perfect square, it must be $3p^2+q = 0$, which is impossible.

Real Numbers

The complete set of rational and irrational numbers is the set of real numbers and is denoted by R. Thus, $R = Q \cup Q^c$. Real numbers can be represented as points of a line. This line is called as real line or number line.

All the real numbers follow the order property i.e. if there are two distinct real numbers a and b then either

$a < b$ or $a > b$



Remarks

1. Integers are rational numbers, but converse need not be true.
2. Negative of an irrational number is an irrational number.
3. Sum of a rational number and an irrational number is always an irrational number e.g. $2+\sqrt{3}$
4. The product of a non zero rational number & an irr. number will always be an irrational number.
5. If $a \in Q$ and $b \notin Q$, then ab is rational number only if $a = 0$.
6. Sum, difference, product and quotient of two irrational numbers need not be an irrational number (it may be a rational number also).

Complex Number

A number of the form $a + ib$ is called complex number, where $a, b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by C.

Remark

1. It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

DIVISIBILITY TEST

1. A number is divisible by 2 iff the digit at the unit place is divisible by 2.
2. A number is divisible by 3 iff the sum of the digits of a number is divisible by 3.
3. A number is divisible by 4 iff last two digits of the number together are divisible by 4.
4. A number is divisible by 5 iff digit at the unit place is either 0 or 5.
5. A number is divisible by 6 iff the digit at the unit place of the number is divisible by 2 & sum of all digits of the number is divisible by 3.
6. A number is divisible by 8 iff the last 3 digits, all together, is divisible by 8.
7. A number is divisible by 9 iff sum of all it's digits is divisible by 9.
8. A number is divisible by 10 iff it's last digit is 0.
9. A number is divisible by 11 iff the difference between the sum of the digits at even places and sum of the digits at odd places is a multiple of 11.
   Example. 1298, 1221, 123321, 12344321, 1234554321, 123456654321, 795432

Example 5

Prove that :

1. The sum ab+ba is multiple of 11.
2. A three-digit number written by one and the same digit is entirely divisible by 37.

Sol. **1.** $ab+ba = (10a + b) + (10b + a) = 11(a + b)$;

2. $aaa = 100a + 10a + a = 111a = 37.3a$

Example 6

If the number A 3 6 4 0 5 4 8 9 8 1 2 7 0 6 4 4 B is divisible by 99 then the ordered pair of digits (A, B) is

Sol. $S_{odd} = A + 37$; $S_{Even} = B + 34 \Rightarrow A - B + 3 = 0$ or 11

and $A + B + 71$ is a multiple of 9

$\Rightarrow A - B = -3$ or 8 and $A + B = 1$ or 10

Ans. : (9, 1)

Example 7

Consider a number N = 2 1 P 5 3 Q 4. Find the number of ordered pairs (P, Q) so that the number 'N is divisible by 44, is

Sol. $S_{odd} = P + 9$, $S_{Even} = Q + 6 \Rightarrow S_{odd} - S_{Even} = P - Q + 3$

'N' is divisible is 11 if $P - Q + 3 = 0, 11$

$P - Q = -3$ ....(i)

or $P - Q = 8$ ....(ii)

N is divisible by 4 if $Q = 0, 2, 4, 6, 8$
From Equation (i)
$Q = 0 \quad P = -3$ (not possible)
$Q = 2 \quad P = -1$ (not possible)
$Q = 4 \quad P = 1 \quad Q = 6 \quad P = 3 \quad Q = 8 \quad P = 5$
$\therefore$ Number of ordered pairs is 3
From equation (ii)
$Q = 0 \quad P = 8 \quad Q = 2 \quad P = 10$ (not possible)
similarly $Q \neq 4, 6, 8$
$\therefore$ No. of ordered pairs is 1
$\therefore$ Total number of ordered pairs, so that number 'N' is divisible by 44, is 4

LCM AND HCF

1. HCF is highest common factor between any two or more numbers (or algebraic expression).
2. LCM is least common multiple between any two or more numbers (or algebraic expression)
3. Multiplication of LCM and HCF of two numbers is equal to multiplication of two numbers.
4. LCM of $\left(\frac{a}{b}, \frac{p}{q}, \frac{\ell}{m}\right) = \frac{\text{LCM of } (a, p, \ell)}{\text{HCF of } (b, q, m)}$
5. HCF of $\left(\frac{a}{b}, \frac{p}{q}, \frac{\ell}{m}\right) = \frac{\text{HCF of } (a, p, \ell)}{\text{LCM of } (b, q, m)}$
6. LCM of rational and irrational number is not defined.

Remainder Theorem

Let P(x) be any polynomial of degree greater than or equal to one and 'a' be any real number. If P(x) is divided by (x–a), then the remainder is equal to P(a).

Factor Theorem

Let P(x) be polynomial of degree greater than or equal to 1 and 'a' be a real number such that P(a) = 0, then (x – a) is a factor of P(x). Conversely, if (x – a) is a factor of P(x), then P(a) = 0.

Some Important Identities

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
2. $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
6. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$
7. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$= a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$
9. $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$
10. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$
If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$
11. $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$
12. $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

Remarks

1. $ab + bc + ca = abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$
2. $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

Definition Of Indices

If 'a' be any non-zero real or imaginary number and m is positive integer than $a^m = a.a.a. ....a$ (m times) where 'a' is base 'm' is index.

Law of Indices

1. $a^0 = 1, \ (a \neq 0)$
2. $a^{-m} = \frac{1}{a^m}, \ (a \neq 0)$
3. $a^{m+n} = a^m . a^n$, where m and n real numbers
4. $a^{m-n} = \frac{a^m}{a^n}$, where m and n real numbers, $a \neq 0$
5. $(a^m)^n = a^{mn}$
6. $a^{p/q} = \sqrt[q]{a^p}$

Example 8

Find p and q so that $(x + 2)$ and $(x - 1)$ may be factors of the polynomial $f(x) = x^3 + 10x^2 + px + q$.

Sol. Since $(x + 2)$ is a factor $\Rightarrow$ $f(-2)$ must be zero

$\therefore -8 + 40 - 2p + q = 0$ **...(1)**

Since $(x - 1)$ is a factor $\Rightarrow$ $f(1)$ must be zero

$\therefore 1 + 10 + p + q = 0$ **...(2)**

From (1) and (2), by solving we get $p = 7$ and $q = -18$

Example 9

Show that $(2x + 1)$ is a factor of the expression $f(x) = 32x^5 - 16x^4 + 8x^3 + 4x + 5$.

Sol. Since $(2x + 1)$ is to be a factor of f(x), $f\left(-\frac{1}{2}\right)$ should be zero.

$$f\left(-\frac{1}{2}\right) = 32\left(-\frac{1}{2}\right)^5 - 16\left(-\frac{1}{2}\right)^4 + 8\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right) + 5 = 0$$

Hence $(2x + 1)$ is a factor of f(x).

Example 10

Without using the Remainder theorem, find the remainder when $f(x) = x^6 - 19x^5 + 69x^4 - 151x^3 + 229x^2 + 166x + 26$ is divided by $x - 15$.

Sol. f(x) can be written as

$(x^6 - 15x^5) - 4(x^5 - 15x^4) + 9(x^4 - 15x^3) - 16(x^3 - 15x^2) - 11(x^2 - 15x) + (x - 15) + 41$

or as $f(x) = x^5 (x - 15) - 4x^4(x - 15) + 9x^3(x - 15) - 16x^2(x - 15) - 11x(x - 15) + (x - 15) + 41$

Since the first six terms have $x - 15$ as a factor, remainder = 41.

Example 11

Without actual division prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

Sol. Let $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$ and $g(x) = x^2 - 3x + 2$ be the given polynomials.

Then $g(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$

In order to prove that f(x) is exactly divisible by g(x), it is sufficient to prove that $x - 1$ and $x - 2$ are factors of f(x). For this it is sufficient to prove that $f(1) = 0$ and $f(2) = 0$.

Now, $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$

$\Rightarrow f(1) = 2 \times 1^4 - 6 \times 1^3 + 3 \times 1^2 + 3 \times 1 - 2$

and, $f(2) = 2 \times 2^4 - 6 \times 2^3 + 3 \times 2^2 + 3 \times 2 - 2$

$\Rightarrow f(1) = 2 - 6 + 3 + 3 - 2$ and $f(2) = 32 - 48 + 12 + 6 - 2$

$\Rightarrow f(1) = 8 - 8$ and $f(2) = 50 - 50$

$\Rightarrow f(1) = 0$ and $f(2) = 0$

$\Rightarrow$ $(x - 1)$ and $(x - 2)$ are factors of f(x)

$\Rightarrow$ $g(x) = (x - 1)(x - 2)$ is a factors of f(x).

Hence, f(x) is exactly divisible by g(x).

Example 12

Using factor theorem, show that $a - b$, $b - c$ and $c - a$ are the factors of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

Sol. By factor theorem, $a - b$ will be a factor of the given expression if it vanishes by substituting $a = b$ in it. Substituting $a = b$ in the given expression,

we have $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

$= b(b^2 - c^2) + b(c^2 - b^2) + c(b^2 - b^2)$

$= b^3 - bc^2 + bc^2 - b^3 + c(b^2 - b^2) = 0$

$\therefore$ $(a - b)$ is a factor of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

Similarly, we can show that $(b - c)$ and $(c - a)$ are also factors of the given expression.

Hence, $(a - b)$, $(b - c)$ and $(c - a)$ are factors of the given expression.

Example 13

Show that $x - 2y$ is a factor of $3x^3 - 2x^2y - 13xy^2 + 10y^3$.

Sol. Let $f(x) = 3x^3 - 2x^2y - 13xy^2 + 10y^3$

Then $f(2y) = 3(2y)^3 - 2y(2y)^2 - 13y^2(2y) + 10y^3 = 24y^3 - 8y^3 - 26y^3 + 10y^3 = 0$

Hence $x - 2y$ is a factor of f(x).

Example 14

Show that $a^n - b^n$ is divisible by $a - b$ if n is any positive integer odd or even.

Sol. Let $a^n - b^n = f(a)$. By Remainder theorem, $f(b) = b^n - b^n = 0$ (replacing a by b)

$\therefore$ $a - b$ is a factor of $a^n - b^n$.

Example 15

Show that $a^n - b^n$ is divisible by $(a + b)$ when n is an even positive integer but not if n is odd.

Sol. Let $a^n - b^n = f(a)$.

Now $f(-b) = (-b)^n - b^n = b^n - b^n = 0$

if n is even and hence $a + b$ is a factor of $a^n - b^n$

If n is odd, $f(-b) = -b^n - b^n = -2b^n \neq 0$.

Example 16

If $a + b + c = 0$, prove that

$a^4 + b^4 + c^4 = 2(b^2 c^2 + c^2 a^2 + a^2 b^2) = 1/2(a^2 + b^2 + c^2)^2$

Sol. We know that,

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow a^2+b^2+c^2 = -2(ab+bc+ca)$ [$\because$ $a+b+c=0$]

Squaring both sides of the relation

$(a^2 + b^2 + c^2)^2 = [-2(bc + ca + ab)]^2$

$= 4\{b^2 c^2 + c^2 a^2 + a^2 b^2\} + 2\{bc.\ ca + ca.\ ab + ab.\ bc\}$,

$= 4(b^2 c^2 + c^2 a^2 + a^2 b^2) + 8abc(a + b + c)$

$= 4(b^2 c^2 + c^2 a^2 + a^2 b^2)$, since $a + b + c = 0$.

Therefore, $2(b^2c^2 + c^2 a^2 + a^2b^2) = 1/2\ (a^2 + b^2 + c^2)^2$.

Also $(a^2 + b^2 + c^2)^2 = (a^4 + b^4 + c^4) + 2(b^2c^2 + c^2 a^2 + a^2b^2)$,

so that $4(b^2c^2 + c^2 a^2 + a^2b^2) = (a^4 + b^4 + c^4) + 2(b^2c^2 + c^2 a^2 + a^2b^2)$

$\Rightarrow a^4 + b^4 + c^4 = 2(b^2c^2 + c^2 a^2 + a^2b^2)$.

Example 17

Solve the equation, $\dfrac{x-ab}{a+b} + \dfrac{x-bc}{b+c} + \dfrac{x-ca}{1+a} = a+b+c$. What happens if $\dfrac{1}{a+b} + \dfrac{1}{b+c} + \dfrac{1}{c+a} = 0$

Sol. $$\left(\frac{x-ab}{a+b} - c\right) + \left(\frac{x-bc}{b+c} - a\right) + \left(\frac{x-ca}{c+a} - b\right) = 0$$

$$\Rightarrow (x - (ab + bc + ca))\left[\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}\right] = 0$$

$\Rightarrow x = ab + bc + ca$.

If $\dfrac{1}{a+b} + \dfrac{1}{b+c} + \dfrac{1}{c+a} = 0$, the given equation becomes an identity & is true for all $x \in R$

RATIO

1. If A and B be two quantities of the same kind, then their ratio is A : B; which may be denoted by the fraction $\dfrac{A}{B}$ (This may be an integer or fraction)

2. A ratio may represented in a number of ways e.g. $\dfrac{a}{b} = \dfrac{ma}{mb} = \dfrac{na}{nb} = \ldots$ where m,n,......are non-zero numbers.

3. To compare two or more ratio, reduce them to common denominator.

4. Ratio between two ratios may be represented as $\dfrac{a}{b} : \dfrac{c}{d} \equiv \dfrac{a/b}{c/d} = \dfrac{ad}{bc}$

5. Ratios are compounded by multiplying them together i.e. $\dfrac{a}{b} \cdot \dfrac{c}{d} \cdot \dfrac{e}{f} \ldots\ldots = \dfrac{ace}{bdf} \ldots\ldots$

6. If a : b is any ratio then its duplicate ratio is $a^2 : b^2$; triplicate ratio is $a^3 : b^3$..... etc.

7. If a : b is any ratio, then its sub-duplicate ratio is $a^{1/2} : b^{1/2}$, sub-triplicate ratio is $a^{1/3} : b^{1/3}$ etc.

PROPORTION

When two ratios are equal, then the four quantities compositing them are said to be proportional.

If $\frac{a}{b}=\frac{c}{d}$, then it is written as a : b = c : d or a : b : : c : d

1. 'a' and 'd' are known as extremes and 'b' and 'c' are known as means.
2. An important property of proportion : Product of extremes = product of means.
3. If a : b = c : d, then b : a = d : c (Invertando)
4. If a : b = c : d, then a : c = b : d (Alternando)
5. If a : b = c : d, then $\frac{a+b}{b}=\frac{c+d}{d}$ (Componendo)
6. If a : b = c : d, then $\frac{a-b}{b}=\frac{c-d}{d}$ (Dividendo)
7. If a : b = c : d, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Componendo and dividendo)
8. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=.....$, then each $=\frac{a+c+e+.....}{b+d+f+......}$ $=\frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$
9. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=..$, then each $=\frac{xa+yc+ze+..}{xb+yd+zf+..}$
10. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=..$, then each $=\left(\frac{xa^n+yc^n+ze^n}{xb^n+yd^n+zf^n}\right)^{1/n}$

Example 18

If $\frac{x+y}{2}=\frac{y+z}{3}=\frac{z+x}{4}$, then find x : y : z.

Sol. Each $=\frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}=$

$$\frac{2(x+y+z)}{9}=\frac{x+y+z}{9/2}$$ and therefore

Each

$$=\frac{(x+y+z)-(y+z)}{\frac{9}{2}-3}=\frac{(x+y+z)-(x+z)}{\frac{9}{2}-4}=\frac{(x+y+z)-(x+y)}{\frac{9}{2}-2}$$

$$=\frac{x}{3/2}=\frac{y}{1/2}=\frac{z}{5/2}\Rightarrow x:y:z=3:1:5$$

Example 19

If a(y + z) = b(z + x) = c(x + y),

then show that $\frac{a-b}{x^2-y^2}=\frac{b-c}{y^2-z^2}=\frac{c-a}{z^2-x^2}$

Sol. Given condition can be written as

$$\frac{y+z}{1/a}=\frac{z+x}{1/b}=\frac{x+y}{1/c}=k \quad ....(1)$$

$$=\frac{(z+x)-(y+z)}{\frac{1}{b}-\frac{1}{a}}=\frac{(x+y)-(x+z)}{\frac{1}{c}-\frac{1}{b}}=\frac{(y+z)-(x+y)}{\frac{1}{a}-\frac{1}{c}}$$

$$=\frac{x-y}{\frac{a-b}{ab}}=\frac{y-z}{\frac{b-c}{bc}}=\frac{z-x}{\frac{c-a}{ca}}=k \quad ....(2)$$

Multiplying (1) and (2), we get,

$$\frac{x^2-y^2}{a-b}=\frac{y^2-z^2}{b-c}=\frac{z^2-x^2}{c-a}$$

$$\Rightarrow\frac{a-b}{x^2-y^2}=\frac{b-c}{y^2-z^2}=\frac{c-a}{z^2-x^2}$$

Example 20

If $x=\frac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$, show that $3bx^2-4ax+3b=0$.

Sol. Taking the left hand side as $\frac{x}{1}$, using componendo and dividendo,

$$\frac{x+1}{x-1}=\frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}$$

Squaring, $\frac{(x+1)^2}{(x-1)^2}=\frac{2a+3b}{2a-3b}$ and again applying componendo and dividendo, We get, $\frac{x^2+1}{2x}=\frac{2a}{3b}$ which gives the answer on cross multiplication.

Example 21

If $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}$,

then show that

$$\frac{9x}{2b+2c-a}=\frac{9y}{2c+2a-b}=\frac{9z}{2a+2b-c}$$

$$\frac{9x}{2b+2c-a}=\frac{9y}{2c+2a-b}=\frac{9z}{2a+2b-c}$$

Sol. Since

$$\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c},$$

each ratio is equal to

$$\frac{2(2z+2x-y)+2(2x+2y-z)-(2y+2z-x)}{2b+2c-a}$$

$$=\frac{9x}{2b+2c-a}$$

Similarly, Each ratio $=\frac{9y}{2c+2a-b}$ and

$\frac{9z}{2a+2b-c}$. Hence proved

Example 22

Solve : $\frac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}-\sqrt{2-x}}=2$

Sol. Writing the R.H.S. as $\frac{2}{1}$ and using componendo and dividendo,

$$\frac{(\sqrt{2+x}+\sqrt{2-x})+(\sqrt{2+x}-\sqrt{2-x})}{(\sqrt{2+x}+\sqrt{2-x})-(\sqrt{2+x}-\sqrt{2-x})}=\frac{2+1}{2-1}$$

(i.e.) $\frac{\sqrt{2+x}}{\sqrt{2-x}}=\frac{3}{1}$

On squaring, We get, $\frac{2+x}{2-x}=\frac{9}{1}$ and again applying componendo and dividendo,

We get, $\frac{4}{2x}=\frac{10}{8}\Rightarrow x=\frac{8}{5}$

INTERVALS

Intervals are subsets of R and generally its used to find domain of inequality. If a and b are two real numbers such that $a < b$ then following types of intervals can be defined Open Interval (a, b) $\{x : a < x < b\}$ i.e. extreme points are not included Closed Interval $[a, b]\{x : a \le x \le b\}$ i.e. extreme points are included It can possible when a and b are finite Semi-Open Interval $(a, b]\{x : a < x \le b\}$ i.e. a is not included and b is included Semi-Closed Interval $[a, b)$ $\{x : a \le x < b\}$ i.e. a is included and b is not included

Method of Intervals

Let $F(x)=(x-a_1)^{k_1}(x-a_2)^{k_2}\,\ldots\ldots\,(x-a_{n-1})^{k_{n-1}}(x-a_n)^{k_n}$.

Here $k_1, k_2, \ldots, k_n \in Z$ are $a_1, a_2, \ldots. a_n$ are fixed real numbers satisfying the condition

$a_1 < a_2 < a_3 < \ldots < a_{n-1} < a_n$

For solving $F(x) > 0$ or $F(x) < 0$, consider the following algorithm:

1. Mark the numbers $a_1, a_2, \ldots. a_n$ on the number axis and put plus sign in the interval on the right of the largest of these numbers,i.e. on the right of a_n.
2. Then put plus sign in the interval on the left of a_n if k_n is an even number and minus sign if k_n is an odd number. In the next interval, we put a sign according to the following rule :
3. When passing through the point a_{n-1},the polynomial $F(x)$ changes sign if k_{n-1} is an odd number. Then consider the next interval and put a sign in it using the same rule.
4. Thus, consider all the intervals. The solution of the inequality $F(x) > 0$ is the union of all intervals in which we put plus sign and the solution of the inequality and $F(x) < 0$ is the union of all intervals in which we put minus sign.

Frequently Used Inequalities

1. $(x-a)(x-b) < 0 \Rightarrow x \in (a,b)$. where $a < b$
2. $(x-a)(x-b) > 0 \Rightarrow x \in (-\infty,a)\cup(b,\infty)$, where $a < b$
3. $x^2 \le a^2 \Rightarrow x \in [-a,a]$
4. $x^2 \ge a^2 \Rightarrow x \in (-\infty,-a]\cup[a,\infty)$
5. $ax^2+bx+c<0$, $(a>0) \Rightarrow x \in (\alpha,\beta)$, where α,β $(\alpha<\beta)$ are the roots of the equation $ax^2+bx+c=0$
6. $ax^2+bx+c>0$, $(a>0) \Rightarrow x \in (-\infty,\alpha)\cup(\beta,\infty)$, where $\alpha,\beta(\alpha<\beta)$ are the roots of the equation $ax^2+bx+c=0$

SECTION - B : LOG & PROPERTIES

LOGARITHM OF A NUMBER

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N.

This number is designated as $\log_a N$.

Hence, $\ell og_a N = x \Leftrightarrow a^x = N, a > 0, \ a \neq 1 \ \& \ N > 0$

Common and natural logarithm

$\ell og_{10} N$ is referred as a common logarithm and $\ell og_e N$ is called as natural logarithm of N to the base Napierian and is popularly written as $\ell n\, N$. Note that e is an irrational quantity lying between 2.7 to 2.8. **Note that** $e^{\ell n\, x} = x$.

The existence and uniqueness of the number $\ell og_a N$ follows from the properties of an exponential functions.

From the definition of the logarithm of the number N to the base 'a', we have an identity :

$a^{\log_a N} = N, a > 0, \ a \neq 1 \ \& \ N > 0$

This is known as the

FUNDAMENTAL LOGARITHMIC IDENTITY.

Other popular Identities

$\ell og_a 1 = 0$	$(a > 0, a \neq 1)$
$\ell og_a a = 1$	$(a > 0, \ a \neq 1)$
$\ell og_{1/a} a = -1$	$(a > 0, a \neq 1)$

Remember

$\ell og_{10} 2 = 0.3010, \ell og_{10} 3 = 0.4771, \ell n 2 = 0.693, \ell n 10 = 2.303$

The principal properties of logarithms :

Let M & N are arbitrary positive numbers, $a > 0, \ a \neq 1, b > 0, b \neq 1$ and α is any real number then ;

1. $\ell og_a (M.N) = \ell og_a M + \ell og_a N$
2. $\ell og_a (M/N) = \ell og_a M - \ell og_a N$
3. $\ell og_a M^\alpha = \alpha. \ \ell og_a M$
4. $\ell og_{a^\beta} M = \dfrac{1}{\beta} \ell og_a M$
5. $\ell og_b M = \dfrac{\ell og_a M}{\ell og_a b}$ (base change theorem)

Remarks

1. $\ell og_b a . \ell og_a b = 1 \Leftrightarrow \ell og_b a = \dfrac{1}{\ell og_a b}$
2. $\ell og_b a. \ \ell og_c b. \ \ell og_a c = 1$
3. $\ell og_y x. \ \ell og_z y. \ \ell og_a z = \ell og_a x.$
4. $e^{\ln a^x} = a^x$.

Example 23

Compute $\sqrt{\left(\dfrac{1}{\sqrt{27}}\right)^{2-\frac{\log_5 13}{2\log_5 9}}}$

Sol. $\dfrac{\log_5 13}{2\log_5 9} = \dfrac{1}{2}\log_9 13$

$= \sqrt{\left(\dfrac{1}{\sqrt{27}}\right)^{2-\frac{\log_5 13}{2\log_5 9}}} = \sqrt{\dfrac{1}{27} \div \left(\dfrac{1}{\sqrt{27}}\right)^{\frac{\log_5 13}{2\log_5 9}}}$

$= \sqrt{\dfrac{1}{27} \times \left(\sqrt{27}\right)^{\frac{1}{2}\log_9 13}} = \sqrt{3^{-3} \times 3^{\frac{3}{8}\log_3 13}}$

$= \sqrt{3^{-3} \times 13^{\frac{3}{8}}} = 3^{\frac{-3}{2}} \times 13^{\frac{3}{16}}$

Example 23

Compute $\log_{ab} \dfrac{\sqrt[3]{a}}{\sqrt{b}}$ if $\log_{ab} a = 4$.

Sol. By the laws of logarithms, we have,

$\log_{ab} \dfrac{\sqrt[3]{a}}{\sqrt{b}} = \dfrac{1}{3}\log_{ab} a - \dfrac{1}{2}\log_{ab} b = \dfrac{4}{3} - \dfrac{1}{2}\log_{ab} b$

Also, $1 = \log_{ab} ab = \log_{ab} a + \log_{ab} b = 4 + \log_{ab} b$

It follows that $\log_{ab} b = -3$ and so

$\log_{ab} \dfrac{\sqrt[3]{a}}{\sqrt{b}} = \dfrac{4}{3} - \dfrac{1}{2}.(-3) = \dfrac{17}{6}$

Example 25

Compute the value of $\dfrac{1}{\log_2 36} + \dfrac{1}{\log_3 36}$.

Sol. $\dfrac{1}{\log_2 36} + \dfrac{1}{\log_3 36} = \log_{36} 2 + \log_{36} 3$

$= \log_{36} 6 = \dfrac{1}{2}$

Example 26

Find the domain of $\log_{x-3}(2x-3)$.

Sol. $x - 3 > 0, x - 3 \neq 1$ and $2x - 3 > 0 \Rightarrow x > 3$, $x \neq 4$ and $x > 3/2 \Rightarrow (3, 4) \cup (4, \infty)$

Example 27

Given $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \dfrac{2}{s^3+1}$.

Write $\log_2 \dfrac{a^2b^5}{c^4}$ as a function of 's' (a, b, c > 0, c ≠ 1).

Sol. Given $\log_2 a = s$ ....(1)

$\log_2 b = 2s^2$ ....(2)

$\log_8 c^2 = \dfrac{s^3+1}{2}$ ....(3)

$\Rightarrow \dfrac{2\log c}{3\log 2} = \dfrac{s^3+1}{2}$

$\Rightarrow 4\log_2 c = 3(s^3+1)$ ....(4)

Hence the value of $2\log_2 a + 5\log_2 b - 4\log_2 c$

$= 2s + 10s^2 - 3(s^3+1)$

Example 28

If log25 = a and log 225 = b, then find the value of $\log\left(\left(\dfrac{1}{9}\right)^2\right) + \log\left(\dfrac{1}{2250}\right)$ in terms of a and b.

Sol. $\log 25 = a$; $\log 225 = b$

$2\log 5 = a$; $\log(25 \cdot 9) = b$

or $\log 25 + 2\log 3 = b$

$\Rightarrow 2\log 3 = b - a$

Now, $\log\left(\dfrac{1}{9}\right)^2 + \log\left(\dfrac{1}{2250}\right)$

$= -2\log 9 - \log 2250$

$= -4\log 3 - [\log 225 + \log 10]$

$= -2(b-a) - [b+1]$

$= -2b + 2a - b - 1$

$= 2a - 3b - 1$

Example 29

Compute $\log_6 16$ if $\log_{12} 27 = a$

Sol. $\log_6 16 = 4\log_6 2 = \dfrac{4}{\log_2 6} = \dfrac{4}{1+\log_2 3}$

Also, $\log_{12} 27 = a = 3\log_{12} 3$

$= \dfrac{3}{\log_3 12} = \dfrac{3}{1+2\log_3 2} = \dfrac{3}{1+\dfrac{2}{\log_2 3}} = \dfrac{3\log_2 3}{2+\log_2 3}$

$\Rightarrow \log_2 3 = \dfrac{2a}{3-a}$

(note that, obviously, $a \neq 3$).

$\Rightarrow \log_6 16 = \dfrac{4(3-a)}{3+a}$.

SECTION - C

LOGARITHMIC EQUATIONS

$\ell og_a x = \ell og_a y$ possible when x = y i.e.

$\ell og_a x = \ell og_a y \Leftrightarrow x = y$

Always check the validity of the given equation i.e. $x > 0$, $y > 0$, $a > 0$, $a \neq 1$

Example 30

For $x \geq 0$, what is the smallest possible value of the expression $\log(x^3 - 4x^2 + x + 26) - \log(x+2)$?

Sol. $\log\dfrac{(x^3-4x^2+x+26)}{(x+2)}$

$= \log\dfrac{(x^2-6x+13)(x+2)}{(x+2)}$

$= \log(x^2 - 6x + 13)$ $[\because x \neq -2]$

$= \log\{(x-3)^2 + 4\}$

$\therefore$ Minimum value is log 4 when x = 3

Example 31

If $\log_6 15 = \alpha$ and $\log_{12} 18 = \beta$ then compute the value of $\log_{25} 24$ in terms of α & β.

Sol. $\alpha = \dfrac{1+\log_3 5}{1+\log_3 2}$; $\beta = \dfrac{2+\log_3 2}{1+2\log_3 2}$

Let $\log_3 2 = x$ and $\log_3 5 = y$

$1 + y = \alpha\,(1 + x)$ ....(1)

$2 + x = \beta\,(2x + 1)$ ....(2)

$\Rightarrow x = \dfrac{2-\beta}{2\beta - 1}$ ....(3)

Putting this value of x in (1)

$y = \dfrac{\alpha\,(1+\beta) - (2\beta - 1)}{2\beta - 1}$ . ...(4)

Now $\log_{25} 24 = \dfrac{3x+1}{2y}$. Substitute the value of x and y to get $\log_{25} 24 = \dfrac{5-\beta}{2\alpha + 2\alpha\beta - 4\beta + 2}$

Example 32

Suppose that *a* and *b* are positive real numbers such that $\log_{27}a + \log_9 b = \dfrac{7}{2}$ and $\log_{27}b + \log_9 a = \dfrac{2}{3}$. Find the value of the *ab*.

Sol. $\log_{27}a + \log_9 b = \dfrac{7}{2} \Rightarrow \dfrac{1}{3}\log_3 a + \dfrac{1}{2}\log_3 b = \dfrac{7}{2}$;

$\log_{27}b + \log_9 a = \dfrac{2}{3} \Rightarrow \dfrac{1}{3}\log_3 b + \dfrac{1}{2}\log_3 a = \dfrac{2}{3}$

Adding both the equations, we get,

$\dfrac{1}{3}\log_3(ab) + \dfrac{1}{2}\log_3(ab) = \dfrac{7}{2} + \dfrac{2}{3} = \dfrac{25}{6}$

$\dfrac{5}{6}\log_3(ab) = \dfrac{25}{6}$

$\Rightarrow \log_3(ab) = 5$

$\Rightarrow ab = 3^5 = 243$

Example 33

If $\log_2\left(\log_2\left(\log_3 x\right)\right) = \log_2\left(\log_3\left(\log_2 y\right)\right) = 0$ then find the value of $(x + y)$.

Sol. $\log_2\left(\log_2\left(\log_3 x\right)\right) = 0$

$\Rightarrow \log_2(\log_3 x) = 1$

$\Rightarrow \log_3 x = 2 \quad \Rightarrow x = 9$

Also, $\log_2\left(\log_3\left(\log_2 y\right)\right) = 0$

$\Rightarrow \log_3(\log_2 y) = 1 \Rightarrow \log_2 y = 3$

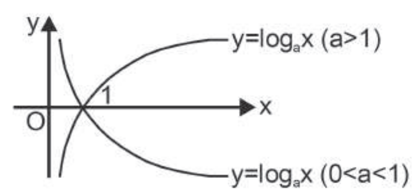
$\Rightarrow y = 8$

$\therefore\ x + y = 17$

SECTION - D : LOG INEQUALITIES

STANDARD LOG INEQUALITIES

1. For $a > 1$ the inequality $0 < x < y$ & $\log_a x < \log_a y$ are equivalent.
2. For $0 < a < 1$ the inequality $0 < x < y$ & $\log_a x > \log_a y$ are equivalent.
3. If $a > 1$ then $\log_a x < p \quad \Rightarrow \quad 0 < x < a^p$
4. If $a > 1$ then $\log_a x > p \quad \Rightarrow \quad x > a^p$
5. If $0 < a < 1$ then $\log_a x < p \Rightarrow \quad x > a^p$
6. If $0 < a < 1$ then $\log_a x > p \Rightarrow \quad 0 < x < a^p$



Remarks

1. If the number & the base are on one side of the unity, then the logarithm is positive; If the number and the base are on different sides of unity, then the logarithm is negative.
2. The base of the logarithm 'a' must not equal to unity.
3. For a non negative number 'a' & $n \geq 2$, $n \in N$

   $\sqrt[n]{a} = a^{1/n}$

Example 34

If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

Sol. First we note that for the functions involved in the given inequality to be defined $(x-1)$ must be greater than 0, that is, $x > 1$.

Now $\log_{0.3}(x-1) < \log_{0.09}(x-1)$

$\Rightarrow \log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

$\Rightarrow \log_{0.3}(x-1)^2 < \log_{0.3}(x-1)$

$\Rightarrow (x-1)^2 > x-1$

[Note that the inequality is reversed because the base of the logarithms lies between 0 and 1]

$\Rightarrow (x-1)^2 - (x-1) > 0$

$\Rightarrow (x-1)(x-2) > 0$ ...(i)

Since $x > 1$,

the inequality (i) will hold if $x > 2$.

Hence x lies in the interval $(2, \infty)$.

Example 35

$x^{\log_5 x} > 5$ then x may belongs to

Sol. We have, $(\log_5 x)^2 > 1$

[taking $\log_5$ both sides]

$\Rightarrow \log_5 x < -1$ or $\log_5 x > 1$

$\Rightarrow x < \frac{1}{5}$ or $x > 5$

But $x > 0 \Rightarrow x \in \left(0, \frac{1}{5}\right) \cup (5, \infty)$

SECTION-E

CHARACTERISTIC & MANTISSA

The common logarithm of a number consists of two parts, integral and fractional, of which the integral part may be zero or an integer (+ve or –ve) and the fractional part a decimal, less than one and always positive.

The integral part is called the *characteristic* and the decimal part is called the *mantissa*. It should be noted that, if the characteristic of the logarithm of N is p then number of significant digit in N = p + 1 if p is the non negative characteristic of log N. Number of zeros after decimal before a significant figure start is –p – 1

Example 36

Let $x = (0.15)^{20}$. Find the characteristic and mantissa in the logarithm of x, to the base 10. Assume $\log_{10}2 = 0.301$ and $\log_{10}3 = 0.477$.

Sol. $\log x = \log(0.15)^{20} = 20 \log\left(\frac{15}{100}\right)$

$= 20[\log 15 - 2]$

$= 20[\log 3 + \log 5 - 2]$

$= 20[\log 3 + 1 - \log 2 - 2] \left[\because \log 5 = \log\frac{10}{2}\right]$

$= 20[-1 + \log 3 - \log 2]$

$= 20[-1 + 0.477 - 0.301]$

$= -20 \times 0.824 = -16.48$

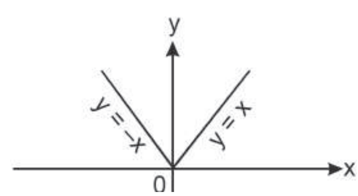
Hence characteristic = –17 and mantissa = 0.52

SECTION - F
MODULUS EQUATIONS / INEQUALITIES

ABSOLUTE VALUE FUNCTION / MODULUS FUNCTION :

A function y = |x| is called the absolute value function or Modulus function. It is defined as :

$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Remarks

1. $|x| < a \Rightarrow -a < x < a \; (a>0)$
2. $|x| > a \Rightarrow x < -a$ or $x > a \; (a>0)$

Example 37

Solution of the equation $|x + 1| - |x - 1| = 3$

Sol. Given, $|x + 1| - |x - 1| = 3$

Case-I

$x < -1$

$-x - 1 + x - 1 = 3$

$\Rightarrow -2 = 3$, which is not possible

Case-II

$-1 \le x < 1$

$x + 1 + x - 1 = 3$

$\Rightarrow \quad x = \frac{3}{2}$ (rejected as $x \in [-1, 1)$)

Case-III

$x \ge 1$

$x + 1 - x + 1 = 3 \Rightarrow 2 = 3$,

which is not possible.

There is no value of x which satisfied the Given equation.

Example 38

If x satisfies $|x-1| + |x-2| + |x-3| \ge 6$, then

Sol. Let $y = |x-1| + |x-2| + |x-3|$

Case-I $x < 1$

$y = 1-x + 2-x + 3-x = 6-3x$

Case-II $1 \le x < 2$

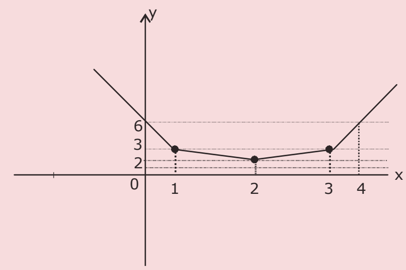
$y = x-1 + 2-x + 3-x = 4-x$

Case-III $2 \le x < 3$

$y = x-1 + x-2 + 3-x = x$

Case-IV $x \ge 3$

$y = x-1 + x-2 + x-3 = 3x-6$



From Graph, one can see that the solution set for $y \ge 6$ is $x \in (-\infty, 0] \cup [4, \infty)$

Exercise 1 (Level-A) JEE Main Level

Basic Maths

1. The value of $0.2\overline{34}$ is -

(A) $\frac{232}{990}$ **(B)** $\frac{232}{9990}$ **(C)** $\frac{232}{900}$ **(D)** $\frac{232}{9909}$

[C. 75.76%, I.C. 6.82%, U.A. 17.42%]

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then find the value of $\frac{2a^4b^2+3a^2c^2-5e^4f}{2b^6+3b^2d^2-5f^5}$ in terms of a and b

(A) $\frac{a^2}{b^2}$ **(B)** $\frac{a^4}{b^4}$ **(C)** $\frac{a^3}{b^3}$ **(D)** $\frac{a^6}{b^6}$

[C. 67.01%, I.C. 28.99%, U.A. 4.00%]

3. If a, b, c are real and distinct numbers, then the value of $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$ is

(A) 1 **(B)** abc **(C)** 2 **(D)** 3

[C. 65.82%, I.C. 29.87%, U.A. 4.31%]

4. If x – a is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to

(A) 0 **(B)** 2 **(C)** – 2 **(D)** 1

[C. 65.60%, I.C. 29.24%, U.A. 5.16%]

5. Express $0.3\bar{6}$ as a fraction in simplest form.

(A) $\frac{11}{30}$ **(B)** $\frac{11}{90}$ **(C)** $\frac{33}{10}$ **(D)** $\frac{10}{33}$

[C. 65.26%, I.C. 17.82%, U.A. 16.92%]

6. If H.C.F. (a, b) = 12 and a × b = 1800, then L.C.M. (a, b) =

(A) 3600 **(B)** 900 **(C)** 150 **(D)** 90

[C. 63.12%, I.C. 33.42%, U.A. 3.46%]

7. If a, b, c are real, then a (a – b) + b (b – c) + c (c – a) = 0, only if

(A) a + b + c = 0

(B) a = b = c

(C) a = b or b = c or c = a

(D) a – b – c = 0

[C. 62.50%, I.C. 31.70%, U.A. 5.80%]

Log Properties

8. If $\log_7(\log_2(\log_\pi x))$ vanishes, then x equals.

(A) π^2 **(B)** 4 **(C)** 49 **(D)** 1

[C. 80.00%, I.C. 12.00%, U.A. 8.00%]

9. $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ is equal to-

(A) abc **(B)** $\frac{1}{abc}$ **(C)** 0 **(D)** 1

[C. 71.45%, I.C. 16.35%, U.A. 12.20%]

10. The value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$ is equal to

(A) 49 **(B)** 625

(C) 216 **(D)** 890

[C. 61.78%, I.C. 32.00%, U.A. 6.22%]

11. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then the value of x is.

(A) 11 **(B)** 10 **(C)** 9 **(D)** 12

[C. 61.71%, I.C. 34.20%, U.A. 4.09%]

12. Simplify the expression $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(A) 1 **(B)** 0

(C) $7^{\log 35}$ **(D)** 5

[C. 58.94%, I.C. 34.64%, U.A. 6.42%]

13. The value of the expression $\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}$ is equal to

(A) 1 **(B)** 1/2 **(C)** 1/3 **(D)** 1/6

[C. 58.47%, I.C. 18.64%, U.A. 22.89%]

14. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(A) 1/2 **(B)** 1 **(C)** 2 **(D)** 4

[C. 58.46%, I.C. 35.51%, U.A. 6.03%]

15. Let $3^a = 4$, $4^b = 5$, $5^c = 6$, $6^d = 7$, $7^e = 8$, and $8^f = 9$. The value of the product (abcdef), is

(A) 1 **(B)** 2 **(C)** $\sqrt{6}$ **(D)** 3

[C. 56.26%, I.C. 36.66%, U.A. 7.08%]

16. If $a = \log_{12} 18$ & $b = \log_{24} 54$, then find the value of $ab + 5(a - b)$.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

[C. 55.20%, I.C. 40.36%, U.A. 4.44%]

17. $2^{(\log_2 4)^2}$ is equal to

(A) 2 **(B)** 4 **(C)** 8 **(D)** 16

[C. 53.48%, I.C. 41.78%, U.A. 4.74%]

18. If x = α satisfies the equation $x^{\log_{\sqrt{x}}(x-2)} = 16$, than α is

(A) even composite

(B) odd composite

(C) rational but not integer

(D) irrational

[C. 53.20%, I.C. 39.20%, U.A. 7.60%]

19. Anti logarithm of 0.75 to the base 16 has the value equal to

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12

[C. 52.99%, I.C. 42.06%, U.A. 4.95%]

Log Equations

20. Solution set of the equation $\log(8 - 10x - 12x^2) = 3\log(2x - 1)$ is

(A) {1} **(B)** {3, 2} **(C)** {5} **(D)** ϕ

[C. 78.82%, I.C. 8.83%, U.A. 12.35%]

21. Find x which satisfy the equation $\log_{x-1} 3 = 2$

(A) $\left(1+\sqrt{3}\right)$ **(B)** 2

(C) $\sqrt{3}$ **(D)** $1-\sqrt{3}$

[C. 68.08%, I.C. 15.96%, U.A. 15.96%]

22. $x^{1+\log x} = 10x$ then the value of x is

(A) 10 **(B)** 11 **(C)** 100 **(D)** 5

[C. 66.39%, I.C. 11.04%, U.A. 22.57%]

23. If $\log_2(\log_3(\log_4(x))) = 0$ and $\log_3(\log_4(\log_2(y))) = 0$ and $\log_4(\log_2(\log_3(z)))=0$ then the sum of x, y and z is

(A) 89 **(B)** 58 **(C)** 105 **(D)** 50

[C. 64.65%, I.C. 11.63%, U.A. 23.72%]

24. If $\log_4 2+\log_4 4+\log_4 x+\log_4 16 = 6$, then x =

(A) 64 **(B)** 4 **(C)** 8 **(D)** 32

[C. 64.32%, I.C. 26.29%, U.A. 9.39%]

25. If $\log_4 \log_3 \log_2 x = 0$. Then x =

(A) 16 **(B)** 8 **(C)** 4 **(D)** 12

[C. 63.78%, I.C. 25.54%, U.A. 10.68%]

26. If $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_3 (x + y) = 2$ then

(A) x = 1, y = 8 **(B)** x = 8, y = 1

(C) x = 3, y = 6 **(D)** x = 9, y = 3

[C. 60.57%, I.C. 12.00%, U.A. 27.43%]

27. If $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 \left(x^2\right) + \log_8 \left(x^3\right) + \log_{16} \left(x^4\right) = 40$ then x is equal to

(A) 8 **(B)** 16 **(C)** 32 **(D)** 256

[C. 60.12%, I.C. 36.90%, U.A. 2.98%]

28. If $\log_2 \log_3 \log_4 \log_5 (A) = x$, then the value of A is

(A) 120^x

(B) 2^{60x}

(C) $2^{3^{4^{5^x}}}$

(D) $5^{4^{3^{2x}}}$

[C. 58.77%, I.C. 40.76%, U.A. 0.47%]

29. $\log_{(5-x)}\{x^2-2x+65\}=2$, Find x

(A) 4 **(B)** −3 **(C)** −5 **(D)** 10

[C. 55.40%, I.C. 12.91%, U.A. 31.69%]

Log Inequalities

30. $x^{\log_5 x}>5$ then x may belongs to

(A) $\left(0,\frac{1}{5}\right)$ **(B)** $(0,\infty)$
(C) $(-\infty,5)$ **(D)** $(1,\infty)$

[C. 74.33%, I.C. 12.91%, U.A. 12.76%]

31. $\log_5(3x-1)<1$

(A) $(1,2)$ **(B)** $\left(\frac{1}{3},2\right)$
(C) $(2,3)$ **(D)** $(1,3)$

[C. 72.10%, I.C. 12.05%, U.A. 15.85%]

32. $\log_{0.5}(x^2-5x+6)>-1$

(A) $(1,3)$
(B) $(1,4)$
(C) $(2,3)$
(D) $(1,2)\cup(3,4)$

[C. 64.27%, I.C. 29.36%, U.A. 6.37%]

33. $\log_{1/4}\left(\frac{35-x^2}{x}\right)\geq-\frac{1}{2}$

(A) $\left(-\sqrt{35},\sqrt{35}\right)$
(B) $(-7,0)\cup(3,\infty)$
(C) $\left[-7,-\sqrt{35}\right)\cup\left[5,\sqrt{35}\right)$
(D) $\left(-\infty,-\sqrt{35}\right)\cup\left(0,\sqrt{35}\right)$

[C. 62.67%, I.C. 29.84%, U.A. 7.49%]

34. $\log_{3x+5}(9x^2+8x+8)>2$

(A) $\left[-\frac{4}{3},\frac{-17}{22}\right]$ **(B)** $\left(\frac{-4}{3},\frac{-17}{22}\right)$
(C) $\left[-\frac{4}{3},\frac{-17}{22}\right)$ **(D)** $\left(-\frac{4}{3},\frac{-17}{22}\right]$

[C. 61.20%, I.C. 28.09%, U.A. 10.70%]

35. $\frac{\log^2 x-3\log x+3}{\log x-1}<1$

(A) $[0,10]$ **(B)** $(0,10)$
(C) $(0,\ 100)$ **(D)** $[0,\ 100]$

[C. 60.33%, I.C. 33.97%, U.A. 5.71%]

Characteristic & Mantissa

36. Let 'm' be the number of digits in 3^{40} and 'p' be the number of zeroes in 3^{-40} after decimal before starting a significant digit the $(m+p)$ is $(\log 3=0.4771)$

(A) 40 **(B)** 39 **(C)** 41 **(D)** 38

[C. 55.03%, I.C. 39.56%, U.A. 5.41%]

37. Number of digits in $N=6^{100}$ (where $\log 2=0.3010$, $\log 3=0.4771$)

(A) 77 **(B)** 78 **(C)** 79 **(D)** 80

[C. 54.19%, I.C. 25.12%, U.A. 20.69%]

38. How many digits are contained in the number 2^{75} ?

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24

[C. 53.85%, I.C. 38.91%, U.A. 7.24%]

39. The number of zeros immediately after the decimal in 3^{-100}

(A) 50 **(B)** 47 **(C)** 48 **(D)** 49

[C. 47.98%, I.C. 33.44%, U.A. 18.58%]

40. Consider the nubmer $N=2^{\log_4 2^{100}}$. Then Number of digits of N before a decimal starts is [Given $\log_{10}2=0.3010$]

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17

[C. 47.83%, I.C. 18.72%, U.A. 33.45%]

Modulas Equation/ Inequalities

41. $\frac{x^2-7x+12}{2x^2+4x+5}>0$ then $x\in R$

(A) (3,4) **(B)** $(-\infty,3)\cup(4,\infty)$
(C) $(-\infty,4]$ **(D)** $(3,\ \infty)$

[C. 76.88%, I.C. 14.21%, U.A. 8.91%]

42. $(x-1)(x-3) > 0$

(A) $(-\infty, 1)$
(B) $(3, \infty)$
(C) $(-\infty, 1) \cup (3, \infty)$
(D) $[1, 3]$

[C. 75.67%, I.C. 15.34%, U.A. 8.99%]

43. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$

(A) $x = 4$ **(B)** $x = -4$ **(C)** $x = 1 + \sqrt{3}$ **(D)** $x = 2$

[C. 71.89%, I.C. 16.59%, U.A. 11.52%]

44. $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1$, then find x?

(A) 3 **(B)** 2 **(C)** 0 **(D)** 1

[C. 66.46%, I.C. 11.08%, U.A. 22.46%]

45. For the inequality $(x+5)(2x-3)^5(-x+7)^3(3x+8)^2 < 0$, x belongs to

(A) $x \in \left(-5, \frac{-8}{3}\right) \cup \left(-\frac{8}{3}, \frac{3}{2}\right) \cup \left(7, \infty\right)$
(B) $x \in \left(-5, \frac{-8}{3}\right) \cup \left(-\frac{8}{3}, \frac{3}{2}\right) \cup (6, \infty)$
(C) $x \in \left(-5, \frac{-8}{3}\right) \cup \left(-\frac{8}{3}, 2\right) \cup (7, \infty)$
(D) $x \in \left(-\infty, -\frac{8}{3}\right) \cup \left(\frac{3}{2}, \infty\right)$

[C. 60.11%, I.C. 28.19%, U.A. 11.70%]

Mixed Problems

46. Which of the following is correct ?

(A) $x^{\log_x y} = y$ is called fundamental logarithmic identity.
(B) If $x > 1$ and $0 < y < 1$ then $\log_y x > 0$
(C) If $0 < x < 1$ and $y > 1$ then $\log_x y > 0$
(D) $\log_6 5 > \log_4 5$

[C. 59.54%, I.C. 37.40%, U.A. 3.05%]

47. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $(x-1)$ and $(x+1)$ the remainders are 5 and 19 respectively. If f(x) is divided by $(x-2)$, then remainder is-

(A) 0 **(B)** 5 **(C)** 10 **(D)** 2

[C. 56.13%, I.C. 34.84%, U.A. 9.03%]

48. If x, y, z are real numbers greater than 1 and 'w' is a positive real number. If $\log_x w = 24$, $\log_y w = 40$ and $\log_{xyz} w = 12$ then $\log_w z$ has the value equal to

(A) $\frac{1}{120}$ **(B)** $\frac{2}{120}$
(C) $\frac{3}{120}$ **(D)** $\frac{5}{120}$

[C. 54.36%, I.C. 36.24%, U.A. 9.40%]

49. The value of the expression $\log_{10}(\tan 6^0) + \log_{10}(\tan 12^0) + \log_{10}(\tan 18^0) + \ldots + \log_{10}(\tan 84^0)$ is

(A) a whole number
(B) an irrational number
(C) a negative integer
(D) a rational number which is not an integer

[C. 52.69%, I.C. 41.85%, U.A. 5.47%]

50. If $a, b, c \in N$ are three consecutive terms of an increasing G.P. If $\log_6 a + \log_6 b + \log_6 c = 6$ and $(b-a)$ is a cube of a natural number then $(a+b+c)$ equals

(A) 100 **(B)** 189 **(C)** 111 **(D)** 122

[C. 51.61%, I.C. 16.94%, U.A. 31.45%]

Exercise 1 (Level-B) — Basic Learning

Basic Maths

1. Find the value of
$49^{(1-\log_7 2)} + 5^{-\log_5 4}$.

2. Resolve the following into factors
(i) $(x-y)^3 - y^3$
(ii) $a^3 - \frac{1}{a^3} + 4$
(iii) $x^3 - 6x^2 + 11x - 6$
(iv) $x^3 - 9x - 10$
(v) $a^2(b-c)+b^2(c-a)+c^2(a-b)$

3. Factorize :-
(i) $1 + x^4 + x^8$
(ii) $x^4 + 4$

4. Which is greater
(a) $\log_2 3$ or $\log_{1/2} 5$
(b) $\log_7 11$ or $\log_8 5$

5. Simplify: $(x^{1/4} + y^{1/4})$:
$$\left(\left(\frac{\sqrt[3]{y}}{y\sqrt{x}}\right)^{3/2} + \left(\frac{x^{-1/2}}{\sqrt[8]{y^3}}\right)^2\right)$$

Log Properties

6. Calculate $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right)-6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$

7. Simplify
(a) $\log_{1/3} \sqrt[4]{729\sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$
(b) $a^{\left(\frac{\log_b(\log_b N)}{\log_b a}\right)}$

8. Given that $\log_2 3 = a$, $\log_3 5 = b$, $\log_7 2 = c$, express the logarithm of the number 63 to the base 140 in terms of a , b & c.

9. Prove that $a^x - b^y = 0$ where $x = \sqrt{\log_a b}$ & $y = \sqrt{\log_b a}$, (where $a > 0, b > 0$ & $a, b \neq 1$).

10. Find the value of the expression
$(\log 2)^3 + \log 8 . \log 5 + (\log 5)^3$

11. (a) Which is smaller ? 2 or $(\log_\pi 2 + \log_2 \pi)$
(b) Prove that $\log_3 5$ and $\log_2 7$ are both irrational.

12. If $\log(4)+\left(1+\frac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3}+27\right)$. Find the value of x (where $x \in N$)

13. If $\log_e \log_5 \left[\sqrt{2x-2}+3\right] = 0$ then find x.

14. (a) If $x = \log_3 4$ and $y = \log_5 3$, find the value of $\log_3 10$ and $\log_3(1.2)$ in terms of x and y.
(b) If $k^{\log_2 5} = 16$, find the value of $k^{(\log_2 5)^2}$.

Log Equations

15. $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, show that $a^a b^b c^c = 1$

16. If $(\log_5(x))^2 + \log_{5x}\left(\frac{5}{x}\right) = 1$,.then find the value of x.

17. If $x^{\frac{\log x+5}{3}} = 10^{5+\log x}$, then the value of x is

18. (a) If $\log_{10}(x^2-12x+36)=2$. Then the value of x is
(b) If $9^{1+\log_3 x} - 3^{1+\log_3 x} - 210 = 0$, then the value of x is.

19. If $\log_3(3^x - 8) = 2 - x$, then value of x =

20. $\log_4(\log_2 x) + \log_2(\log_4 x) = 2$ then find the value of x.

21. Simplify

$$\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - \left(125\right)^{\log_{25} 6}\right)$$

22. Simplify $5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}}\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_{\frac{1}{2}}\left(\frac{1}{10+2\sqrt{21}}\right)$

23. If
$\log_b a . \log_c a + \log_a b . \log_c b + \log_a c . \log_b c = 3$ (where a, b, c are different positive real numbers $\neq 1$), then find the value of abc.

Log Inequalities

24. Solve the inequality
$\log_{1/3}\left(5x - 1\right) > 0$

25. Solve: $\log_7 \frac{2x-6}{2x-1} > 0$

26. Solve the inequalities
$\log_{1/6}\left(x^2 - 3x + 2\right) + 1 < 0$

Characteristic & Mantissa

27. Let $x = (0.15)^{20}$. Find the characteristic and mantissa of the logarithm of x to the base 10. Assume $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$.

28. Find the number of zeros after decimal before a significant figure in
(i) 3^{-50} (ii) 2^{-100} (iii) 7^{-100}

Modulas Equation/ Inequalities

29. Solve the following linear equations

(i) $|x| + 2 = 3$
(ii) $|x| - 2x + 5 = 0$
(iii) $x|x| = 4$

30. Solve the inequality: $|5 - 2x| < 1$

31. Solve the inequalities
$|2x - 4| < x - 1$

Mixed Problems

32. If
${}_a\log_b c = 3 . 3\log_4 3 . 3^{\log_4 (3)^{\log_4 (3)}} . 3^{\log_4 (3)^{\log_4 (3)^{\log_4 (3)}}} \ldots\ldots\infty$
where a,b,c $\in Q$ then the value of abc is

33. If $x = \log_{2a}\left(\frac{bcd}{2}\right)$, $y = \log_{3b}\left(\frac{acd}{3}\right)$, $z = \log_{4c}\left(\frac{abd}{4}\right)$, $w = \log_{5d}\left(\frac{abc}{5}\right)$
Then Prove that
$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} + \frac{1}{w+1} = \log_{abcd} 120 + 1$

34. Prove that
$\log_5 \sqrt{5\sqrt{5\sqrt{5 \ldots\ldots\ldots\ldots\ldots \infty}}} = 1$

Exercise 2 — JEE Advanced Level

Basic Maths (Single Correct)

1. The expression
$x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\infty}}}$
simplifies to

(A) 1 **(B)** $\frac{-1}{2}$ **(C)** – 1 **(D)** $\frac{1}{2}$

[C. 62.50%, I.C. 30.98%, U.A. 6.52%]

2. If the polynomial $(2x^3 + ax^2 + 3x - 5)$ and $(x^3 + x^2 - 2x + a)$ leave the same remainder when divided by $(x - 2)$, then the value of a is

(A) 2 **(B)** –2 **(C)** 3 **(D)** –3

[C. 62.26%, I.C. 29.62%, U.A. 8.11%]

3. If $x = 2 + 2^{2/3} + 2^{1/3}$ then the value of $x^3 - 6x^2 + 6x$ is

(A) 3 **(B)** 2 **(C)** 1 **(D)** -2

[C. 61.59%, I.C. 34.29%, U.A. 4.13%]

(Multiple Correct)

4. If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, then $\frac{(a^k+b^k+c^k)^{\frac{1}{k}}}{(d^k+e^k+f^k)^{\frac{1}{k}}}$ is equal to :(k∈n)

(A) $\frac{a}{d}$ **(B)** $\frac{b}{e}$ **(C)** $\frac{c}{f}$ **(D)** $\frac{a^3}{d^2}$

[C. 75.97%, I.C. 20.54%, U.A. 3.49%]

Log Properties (Single Correct)

5. Greatest integer less than or equal to the number
$\log_2(15)\ .\ \log_{\frac{1}{6}}(2)\ .\ \log_3\left(\frac{1}{6}\right)$ is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1

[C. 49.26%, I.C. 46.44%, U.A. 4.30%]

6. Which one of the following denotes the greatest positive proper fraction ?

(A) $\left(\frac{1}{4}\right)^{\log_2 6}$ **(B)** $\left(\frac{1}{3}\right)^{\log_3 5}$

(C) $3^{-\log_3 2}$ **(D)** $8^{\left(\frac{1}{-\log_3 2}\right)}$

[C. 48.03%, I.C. 49.16%, U.A. 2.81%]

7. If p, q ∈ N satisfy the equation $x^{\sqrt{x}} = \left(\sqrt{x}\right)^x$ then p & q are -

(A) Prime Numbers **(B)** twin prime

(C) Even number

(D) if $\log_q p$ is defined then $\log_p q$ is not & vice versa

[C. 44.30%, I.C. 28.33%, U.A. 27.37%]

8. The equation
$\sqrt{1+\log_x \sqrt{27}}\,\log_3 x + 1 = 0$ has

(A) one integral solution

(B) one irrational solution

(C) two real solution

(D) no prime solution

[C. 35.45%, I.C. 46.93%, U.A. 17.62%]

9. Let M denote anti $\log_{32} 0.6$ and N denote the value of
$49^{(1-\log_7 2)} + 5^{-\log_5 4}$. Then M.N is :

(A) 100 **(B)** 400 **(C)** 50 **(D)** 200

[C. 34.97%, I.C. 61.63%, U.A. 3.40%]

(Multiple Correct)

10. Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is-

(A) a natural number

(B) a prime number

(C) a rational number

(D) an integer number

[C. 81.40%, I.C. 12.89%, U.A. 5.71%]

11. Which of the following when simplified, vanishes ?

(A) $\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8}$

(B) $\log_2\left(\frac{2}{3}\right) + \log_4\left(\frac{9}{4}\right)$

(C) $-\log_8 \log_4 \log_2 16$

(D) $\log_{10}\cot 1° + \log_{10}\cot 2° + \log_{10}\cot 3° + \ldots\ldots\ldots + \log_{10}\cot 89°$

[C. 71.43%, U.A. 28.57%]

12. Let $a = \log 3$, $b = \frac{\log 3}{\log(\log 3)}$ (all logarithms on base 10) the number a^b is

(A) an odd integer
(B) an even integer
(C) a prime number
(D) a composite number

[C. 66.67%, I.C. 29.67%, U.A. 3.66%]

13. Which of the following real numbers are non-negative?

(A) $\log_5 \sqrt{25.\sqrt[4]{8^{\frac{-5}{3}}.4^{\frac{-3}{2}}}}$
(B) $\log_{\cos\frac{7\pi}{4}}\left(\sin\frac{5\pi}{6}\right)$
(C) $\log_{\tan\frac{4\pi}{3}}\left(\cot\frac{7\pi}{6}\right)$
(D) $\log_2 \sqrt{9.\sqrt[3]{27^{\frac{-5}{3}}.243^{\frac{-7}{5}}}}$

[C. 39.64%, U.A. 60.36%]

Question No. 14 - 16

Paragraph:

A denotes the product xyz where x, y and z satisfy
$\log_3 x = \log 5 - \log 7$
$\log_5 y = \log 7 - \log 3$
$\log_7 z = \log 3 - \log 5$
B denotes the sum of square of solution of the equation
$\log_2(\log_2 x^6 - 3) - \log_2(\log_2 x^4 - 5) = \log_2 3$
C denotes characteristic of logarithm
$\log_2(\log_2 3) - \log_2(\log_4 3) + \log_2(\log_4 5) - \log_2(\log_6 5) + \log_2(\log_6 7) - \log_2(\log_8 7)$

14. The value of $\log_2 A + \log_2 B + \log_2 C$ is equal to

[C. 2.63%, I.C. 23.03%, U.A. 74.34%]

15. The value of A + B + C is equal to

[C. 1.79%, I.C. 26.86%, U.A. 71.35%]

16. The value of | A – B + C | is equal to

(A) – 30 **(B)** 32
(C) 28 **(D)** 30

[C. 0.77%, I.C. 28.84%, U.A. 70.39%]

Log Equations (Single Correct)

17. Number of real solution of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$

(A) none **(B)** exactly 1
(C) exactly 2 **(D)** 4

[C. 52.70%, I.C. 42.70%, U.A. 4.60%]

18. If x_1 and x_2 are solutions of the equation
$\log_5\left(\log_{64}|x| + (25)^x - \frac{1}{2}\right) = 2x$, then

(A) $x_1 = 2x_2$ **(B)** $x_1 + x_2 = 0$
(C) $x_1 = 3x_2$ **(D)** $x_1x_2 = 64$

[C. 47.57%, I.C. 49.51%, U.A. 2.91%]

19. Number of real solution (x) of the equation $|x-3|^{3x^2-10x+3} = 1$ is

(A) exactly four **(B)** exactly three
(C) exactly two **(D)** exactly one

[C. 44.15%, I.C. 50.85%, U.A. 5.00%]

20. $2\log_4(4-x) = 4 - \log_2(-2-x)$. Find the value of x.

(A) −4 **(B)** 4 **(C)** 3 **(D)** -3

[C. 38.60%, I.C. 23.98%, U.A. 37.43%]

(Multiple Correct)

21. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(A) one irrational solution
(B) no prime solution
(C) two real solutions
(D) one integral solution

[C. 79.25%, I.C. 18.04%, U.A. 2.71%]

22. The equation

$x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution
(B) at least one real solution
(C) exactly one irrational solution
(D) complex roots

[C. 76.35%, I.C. 20.36%, U.A. 3.29%]

23. For the equation $\log_{100}|x+y| = \frac{1}{2}$ and $\log_{10} y - \log_{10}|x| = \log_{100} 4$, (x, y) is

(A) (10/3, 20/3) **(B)** (10, 20)
(C) (–10, 20) **(D)** (-10/3, –20/3)

[C. 61.65%, I.C. 36.02%, U.A. 2.33%]

24. If $\log_7 x + \log_{13} x = 1$ and $x = 13^{\log_k 7}$ then k is divisible by

(A) 7 **(B)** 13 **(C)** 17 **(D)** 119

[C. 56.02%, I.C. 9.52%, U.A. 34.46%]

25. The equation $\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has -

(A) no integral solution
(B) one natural solution
(C) two real solution
(D) one irrational solution

[C. 42.61%, I.C. 19.89%, U.A. 37.50%]

(Column Match)

26. Match the following :

Column –I	Column –II
(A) Solution of equation $\frac{\log_2(9-2^x)}{3-x} = 1$ is	(P) 2
(B) The value of expression $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi}$ is greater than	(Q) 1
(C) Let $3^a = 4$, $4^b = 5$, $5^c = 6$, $6^d = 7$, $7^e = 8$ and $8^f = 9$. The value of product (abcdef) is	(R) 3
(D) If $(\log_2(\log_2(\log_3 x))) = \log_2(\log_3(\log_2 y)) = 0$ then value of $x - y$ is	(S) 0

(A) $A \to RS$, $B \to PQS$, $C \to P$, $D \to Q$
(B) $A \to R$, $B \to PQR$, $C \to Q$, $D \to P$
(C) $A \to P$, $B \to PRS$, $C \to S$, $D \to R$
(D) $A \to Q$, $B \to QRS$, $C \to R$, $D \to S$

[C. 56.77%, I.C. 20.65%, U.A. 22.58%]

Log Inequalities (Single Correct)

27. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the number of digits in 6^{15} are-

(A) 15 **(B)** 12
(C) 13 **(D)** 11

[C. 56.23%, I.C. 36.84%, U.A. 6.93%]

28. Solve $\log_{(x+3)}(x^2 - x) < 1$

(A) $x \in [-3, -2] \cup (-1, 0) \cup (1, 2)$
(B) $x \in (-3, -2) \cup (-1, 0) \cup (1, 3)$
(C) $x \in (-3, -2) \cup [-1, 0) \cup (1, 2]$
(D) $x \in (-3, -2] \cup (-1, 0] \cup (1, 3]$

[C. 50.85%, I.C. 42.54%, U.A. 6.61%]

(Multiple Correct)

29. If $\frac{1}{2} \le \log_{0.1} x \le 2$ then

(A) The maximum value of x is $1/\sqrt{10}$
(B) x lies between 1/100 and $1/\sqrt{10}$
(C) x does not lie between 1/100 and $1/\sqrt{10}$
(D) The minimum value of x is 1/100

[C. 54.90%, I.C. 7.73%, U.A. 37.37%]

30. Solution set of the inequality

$(\log_2 x)^4 - \left[\log_{\frac{1}{2}}\left(\frac{x^3}{8}\right)\right]^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4\left(\log_{\frac{1}{2}} x\right)^2$

is $(a, b) \cup (c, d)$ then the correct statement is

(A) $a = 2b$ and $d = 2c$
(B) $b = 2a$ and $d = 2c$
(C) $\log_c d = \log_b a$
(D) there are 4 integers in (c, d)

[C. 46.09%, U.A. 53.91%]

Characteristic & Mantissa (Single Correct)

31. If p is the number of natural numbers whose logarithms to the base 10 have the characteristic p and q is the number of natural numbers logarithms of whose reciprocals to the base 10 have the characteristic - q then $\log_{10} p - \log_{10} q$ has the value equal to

(A) $p + q - 1$ **(B)** $p - q + 1$
(C) $p + q + 1$ **(D)** $p + q$

[C. 45.43%, I.C. 48.47%, U.A. 6.10%]

32. Number of zeros after decimal before a significant figure comes in the number $\left(\frac{5}{6}\right)^{100}$, is

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7

[C. 21.74%, I.C. 48.55%, U.A. 29.71%]

(Multiple Correct)

33. Let $N = 10^{3\log 2 - 2\log(\log 10^3) + \log\left((\log 10^6)^2\right)}$ where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is not equal to

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

[C. 56.86%, I.C. 41.18%, U.A. 1.96%]

34. The number of odd positive integers, the logarithm of whose reciprocal to the base 3 have characterstic – 5, is less then

(A) 81 **(B)** 82 **(C)** 161 **(D)** 162

[C. 55.32%, I.C. 42.55%, U.A. 2.13%]

Modulas Equation/ Inequalities (Single Correct)

35. $|x - 3| + 2|x + 1| = 4$

(A) -1 **(B)** 1 **(C)** 0 **(D)** 5

[C. 53.78%, I.C. 40.47%, U.A. 5.75%]

36. If $|x - 4| + |x| < 3$ then

(A) $x \in \left[\frac{7}{2}, 4\right)$ **(B)** $x \in \left[\frac{7}{2}, 4\right]$
(C) $x \in \left(\frac{7}{2}, 4\right)$ **(D)** $x \in \phi$

[C. 47.48%, I.C. 20.45%, U.A. 32.07%]

37. If x satisfies $|x - 1| + |x - 2| + |x - 3| \geq 6$, then

(A) $0 \leq x \leq 4$ **(B)** $x \leq -2$ or $x \geq 4$
(C) $x \leq 0$ or $x \geq 4$ **(D)** $1 \leq x \leq 3$

[C. 46.05%, I.C. 48.27%, U.A. 5.68%]

38. Number of solutions of $|x - 1| + |x - 2| + |x - 4| + |x - 6| + |x - 7| = 2x + 10$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

[C. 18.90%, I.C. 48.03%, U.A. 33.07%]

Mixed Problems (Single Correct)

39. The number of ordered pairs (x, y) satisfying the equation $x - y = (\log_2 y - \log_2 x)(2 + xy)$ and $x^3 + y^3 = 16$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0

[C. 33.33%, I.C. 62.55%, U.A. 4.12%]

40. The number of integral solutions (x, y) of two equations $\log_x y + \log_y x = y$ and $x^2 + y = 6$ is

(A) 0 **(B)** 1
(C) 2 **(D)** Infinite

[C. 15.99%, I.C. 23.39%, U.A. 60.62%]

(Multiple Correct)

41. Which of the following statement(s) is/are true ?

(A) $\log_{10} 2$ lies between $\frac{1}{4}$ and $\frac{1}{3}$
(B) $\log_{\operatorname{cosec}\left(\frac{\pi}{6}\right)}\left(\cos\frac{\pi}{3}\right) = -1$
(C) $e^{\ln(\ln 3)}$ is smaller than 1
(D) $\log_{10} 1 + \frac{1}{2}\log_{10} 3 + \log_{10}\left(2 + \sqrt{3}\right) = \log_{10}\left(1 + \sqrt{3} + \left(2 + \sqrt{3}\right)\right)$

[C. 32.68%, I.C. 58.69%, U.A. 8.63%]

42. If $x > 1 > y > 0$ then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be

(A) 4 **(B)** 2 **(C)** 1 **(D)** 0

[C. 28.39%, I.C. 60.28%, U.A. 11.33%]

Exercise 3 — Numerical Type

1. Let A denotes the value of

$$\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)+\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$$

when a = 43 and b = 57 and B denotes the value of the expression $\left(2^{\log_6 18}\right)\cdot\left(3^{\log_6 3}\right)$. Find the value of $(A \cdot B)$.

2. If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, Find the value of $\frac{1+xyz}{2yz}$

3. The value of $\log_3 4.\log_4 5.\log_5 6.\log_6 7.\log_7 8.\log_8 9$ is equal to -

4. If $5^{\log x} - 3^{(\log x)-1} = 3^{(\log x)+1} - 5^{(\log x)-1}$, (where Base of log is 10) then $\frac{x}{20}$ is

5. If $\log_x \log_{18}\left(\sqrt{2}+\sqrt{8}\right)=\frac{1}{3}$. Then the value of 8x is equal to ______ .

6. If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$.
The value of
$\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ – 460 equals.

7. Number of real values of x satisfying the equation $\log_2\left(\sqrt{x}+1\right) = \log_3\left(1-\sqrt{x}\right)$ is equal to

8. Let y = $\sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$
Find $y \in N$

9. Find the square of the sum of the roots of the equation.
$\log_3 x . \log_4 x . \log_5 x = \log_3 x . \log_4 x + \log_4 x . \log_5 x + \log_5 x . \log_3 x$

10. Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1, such that $2(\log_a c + \log_b c) = 9\log_{ab} c$.
Find the largest possible value of $\log_a b$.

11. If x , y > 0, $\log_y x + \log_x y = \frac{10}{3}$ and xy = 144, then $\frac{x+y}{2} = \sqrt{N}$ where N is a natural number, find the value of N.

12. Let $(x-5)^{(x-7)} = 1$ and sum of possible values of x be λ then $\frac{\lambda}{13}$ is

13. Find the value of x of the equation, $5^{\log_{10} x} = 50 - x^{\log_{10} 5}$ is ______ .
(Divide your answer by 10)

14. $x^{\log_e\left(\frac{y}{z}\right)} y^{\log_e\left(\frac{z}{x}\right)} z^{\log_e\left(\frac{x}{y}\right)}$ is equal to

15. The value of x satisfying the equation $\log_{10}(2^x + x - 41) = x(1 - \log_{10} 5)$ is ______ .

16. The solution of $\log_2(4 \times 3^x - 6) - \log_2(9^x - 6) = 1$ is

17. The value of x satisfying the equation $9^{\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$ is ______ .

18. Solve for x : $\log_5 120 + (x - 3) - 2.\log_5(1 - 5^{x-3}) = -\log_5(0.2 - 5^{x-4})$.

19. If the value of $x = k$, then find the value of $\left|\frac{1}{k}\right|$ satisfying equation: $\log_{(2x+3)}\left(6x^2 + 23x + 21\right) = 4 - \log_{(3x+7)}\left(4x^2 + 12x + 9\right)$

20. If $5^x \cdot \sqrt[x]{8^{x-1}} = 500$, then the value of x is.

21. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of `x' satisfying the equation is

22. If $x, y \in R$ satisfy simultaneously the equation $\log x + \frac{\log(xy^8)}{(\log x)^2 + (\log y)^2} = 2$

and

$\log y + \frac{\log\left(\frac{x^8}{y}\right)}{(\log x)^2 + (\log y)^2} = 0$. Compute $\frac{xy}{2}$

.

23. Given that $\log(2) = 0.3010$, then the number of digits in the number 2000^{2000} is
(Divide your answer by 100)

24. If $\log_{3x}45 = \log_{4x}40\sqrt{3}$ then the characteristic of x^3 to the base 7 is

25. Let ABC be a triangle right angled at C. The value of $\frac{\log_{b+c} a + \log_{c-b} a}{\log_{b+c} a . \log_{c-b} a}(b + c \neq 1, c - b \neq 1)$ equals

26. If $\log_2(a + b) + \log_2(c + d) \geq 4$. Then the minimum value of the expression $a + b + c + d$ is

27. If x_1 and x_2 are the two solutions of the equation $3^{\log_2 x} - 12x^{\log_{16} 9} = \log_3\left(\frac{1}{3}\right)^{3^3}$, then find the value of $\frac{1}{16}\left(x_1^2 + x_2^2\right)$.

28. Find the integral value of x satisfying the equation $\left|\log_{\sqrt{3}} x - 2\right| - |\log_3 x - 2| = 2$.

Exercise 4 (Level-A) JEE Main (Previous Year Questions)

1. The number of real roots of the equation $5+|2^x-1| = 2^x(2^x-2)$ is :

(A) 2 **(B)** 1 **(C)** 3 **(D)** 4

[C. 31.01%, I.C. 43.60%, U.A. 25.39%] (JEE Main 2019)

2. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2,\ (x>0)$ is equal to :

(A) 12 **(B)** 9 **(C)** 10 **(D)** 4

[C. 24.01%, I.C. 54.61%, U.A. 21.38%] (JEE Main 2019)

3. The number of solutions of the equation $\log_4(x-1)=\log_2(x-3)$ is ______.

[C. 21.03%, I.C. 36.69%, U.A. 42.28%] (JEE Main 2021)

4. The number of significant figures in 50000.020×10^{-3} is _______.

[C. 14.21%, I.C. 43.16%, U.A. 42.63%] (JEE Main 2021)

5. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is

(A) 5 **(B)** 7 **(C)** 8 **(D)** 6

[C. 12.53%, I.C. 58.54%, U.A. 28.93%] (JEE Main 2023)

6. For a natural number n, let $\alpha_n = 19^n - 12^n$. then the value of $\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}$ is ....

[C. 11.98%, I.C. 17.05%, U.A. 70.97%] (JEE Main 2022)

7. $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by

(A) 34 but not by 14

(B) 14 but not by 34

(C) both 14 and 34

(D) neither 14 nor 34

[C. 10.23%, I.C. 55.85%, U.A. 33.92%] (JEE Main 2023)

8. If $a+b+c=1$, $ab+bc+ca=2$ and $abc=3$, then the value of $a^4+b^4+c^4$ is equal to __________.

[C. 8.35%, I.C. 39.50%, U.A. 52.14%] (JEE Main 2021)

9. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0,\ x>0$, is _______.

[C. 6.92%, I.C. 18.58%, U.A. 74.51%] (JEE Main 2021)

10. Let a,b,c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$ then $6a+5bc$ is equal to

[C. 1.24%, I.C. 30.71%, U.A. 68.05%] (JEE Main 2023)

Exercise 4 (Level-B) JEE Advanced (Previous Year Questions)

1. The value of

$$6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}}\right)$$

is

[C. 28.57%, I.C. 37.23%, U.A. 34.20%] (JEE Adv. 2013)

2. Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln(2)} = (3y)^{\ln(3)}$, $3^{\ln(x)} = 2^{\ln(y)}$, Then x_0 is

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 6

[C. 22.21%, I.C. 36.58%, U.A. 41.21%] (JEE Adv. 2011)

3. The value of

$$\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$$

is_____

[C. 10.39%, I.C. 18.10%, U.A. 71.51%] (JEE Adv. 2018)

Answer Key

Exercise 1 (Level-A) — JEE Main Level

1. A	**2.** B	**3.** D	**4.** C	**5.** A	**6.** C	**7.** B	**8.** A
9. D	**10.** D	**11.** B	**12.** B	**13.** D	**14.** B	**15.** B	**16.** B
17. D	**18.** A	**19.** C	**20.** D	**21.** A	**22.** A	**23.** A	**24.** D
25. B	**26.** C	**27.** D	**28.** D	**29.** C	**30.** A	**31.** B	**32.** D
33. C	**34.** B	**35.** B	**36.** B	**37.** B	**38.** C	**39.** B	**40.** C
41. B	**42.** C	**43.** B	**44.** A	**45.** A	**46.** A	**47.** C	**48.** B
49. A	**50.** B						

Exercise 1 (Level-B) — Basic Learning

1. $\frac{25}{2}$

2. (i) $\left(x-2y\right)\left(x^2-xy+y^2\right)$
$\left(ii\right)$ $\left(a-\frac{1}{a}+1\right)\left(a^2+\frac{1}{a^2}-a+\frac{1}{a}+2\right)$
(iii) $(x-1)(x-2)(x-3)$
$\left(iv\right)$ $\left(x+2\right)\left(x^2-2x-5\right)$
(v) $-(a-b)(b-c)(c-a)$

3. (i) $\left(x^4-x^2+1\right)\left(x^2-x+1\right)\left(x^2+x+1\right)$
(ii) $\left(x^2-2x+2\right)\left(x^2+2x+2\right)$

4. $\left(a\right)$ $\log_2 3>\log_{1/2}5$ (b) $\log_7 11>\log_8 5$

5. xy

6. 9

7. (a) = -1 (b) $\log_b N$

8. $\frac{1+2ac}{1+abc+2c}$

10. 1

11. (a) $(\log_2\pi+\log_\pi 2)>2$

12. ϕ

13. 3

14. $\left(a\right)$ $\log_3\left(10\right)=\frac{xy+2}{2y},\ \log_3\left(1.2\right)=\frac{2y+xy-2}{2y}$
$\left(b\right)$ 625

16. $x=1,\ 5,\ \frac{1}{25}$

17. $x=10^3$ or $x=10^{-5}$

18. (a) x = - 4, 16 (b) 5

19. 2

20. 16

21. 1

22. 6

23. 1

24. $\left(\frac{1}{5},\frac{2}{5}\right)$

25. $\left(-\infty, \frac{1}{2}\right)$

26. $(-\infty, -1) \cup (4, \infty)$

27. characteristic = -17 and mantissa = 0.52

28. (i) 23 (ii) 30 (iii) 84

29. (i) $x = \pm 1$ (ii) $x = 5$ (iii) $x = 2$

30. $(2, 3)$

31. $\left(\frac{5}{3}, 3\right)$

32. 16

Exercise 2 — JEE Advanced Level

1. C	**2.** D	**3.** B	**4.** A,B,C	**5.** C	**6.** C	**7.** D	**8.** D
9. A	**10.** A,B,C,D	**11.** A,B,C,D	**12.** A,C	**13.** A,B,C	**14.** 5	**15.** 34	**16.** D
17. C	**18.** B	**19.** B	**20.** A	**21.** A,B,C,D	**22.** A,B,C,D	**23.** A,C	**24.** A,B
25. B,C	**26.** A	**27.** B	**28.** B	**29.** A,B,D	**30.** B,C	**31.** B	**32.** D
33. A,C,D	**34.** B,C,D	**35.** A	**36.** D	**37.** C	**38.** C	**39.** A	**40.** B
41. A,B,D	**42.** B,C,D						

Exercise 3 — Numerical Type

1. 12	**2.** 1	**3.** 2	**4.** 5	**5.** 1	**6.** 9	**7.** 1	**8.** 6
9. 3721	**10.** 2	**11.** 507	**12.** 1	**13.** 10	**14.** 1	**15.** 41	**16.** 1
17. 2	**18.** 1	**19.** 4	**20.** 3	**21.** 3	**22.** 5	**23.** 66.03	**24.** 2
25. 2	**26.** 8	**27.** 17	**28.** 9				

Exercise 4 (Level-A) — JEE Main (Previous Year Questions)

1. B	**2.** C	**3.** 1	**4.** 7	**5.** D	**6.** 4	**7.** A	**8.** 13
9. 1	**10.** 8						

Exercise 4 (Level-B) — JEE Advanced (Previous Year Questions)

1. 4	**2.** C	**3.** 8

POINTS TO REMEMBER

POINTS TO REMEMBER

POINTS TO REMEMBER